Note Title 17.08.2021

The geometry of diffusion tensor imaging

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(based on joint work with: Sumit Kaushik
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⋮)

0. Background

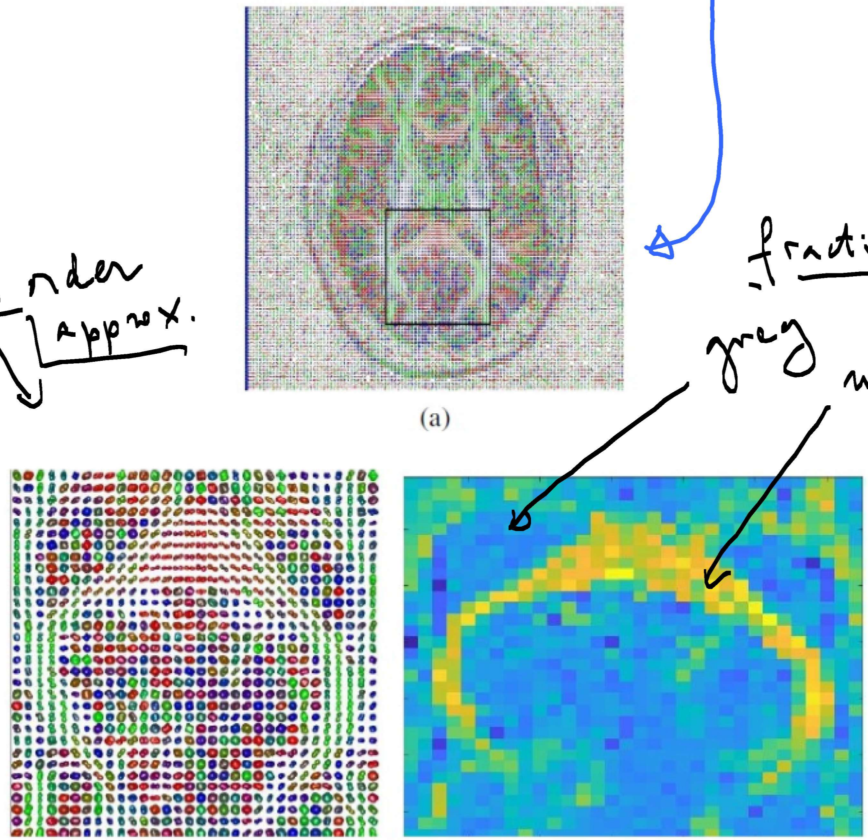
- Diffusion imaging:
  - very strong magnetic field
  - exact frequency to force the water molecules to resonate
  - for each voxel "shoot" in "all" directions => measuring their diffusion

White matter = ANISOTROPIC :

Grey matter = ISOTROPIC

0. Background

Corpus Callosum



(a)

(b) (c)

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Main tasks : Segmentation, Fibre tracking, Biomarkers

Main tools: analysis of tensors

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simplest modality : DTI

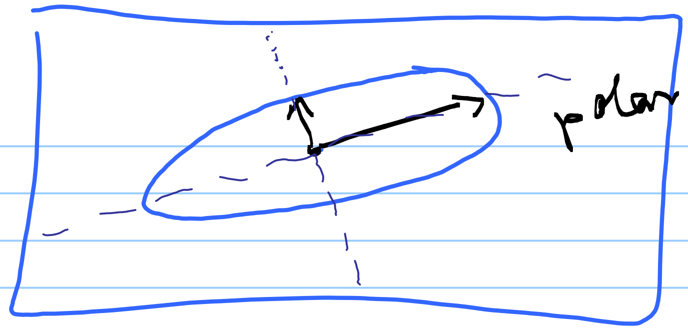
assumes Gaussian character of the diffusion

$\Longrightarrow$ the diffusion speed is approximated by quadratic forms in $\mathbb{R}^3$:

SPD = $\begin{cases} \text{symmetric} \\ \text{positive} \\ \text{definite} \end{cases}$

$$ax^2+by^2+cz^2+2dxy+2exz+2fyz = \begin{pmatrix} x & y & z \end{pmatrix} \cdot \begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

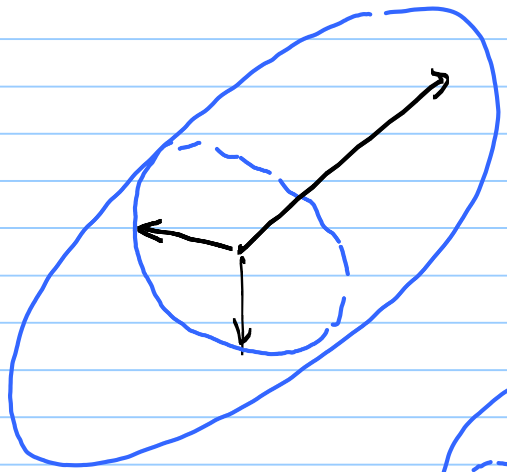
plane:



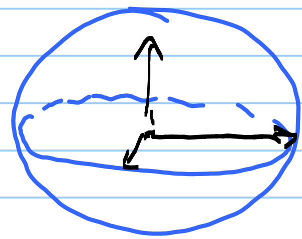
polar coordinates

$$D(x,y) = ax^2 + by^2 = 1$$

space:



$$D(x,y,z) = ax^2 + by^2 + cz^2 = 1$$



isotropic case
$\sim$ sphere

More generally: the Stejskal-Tanner model

$r \in \mathbb{R}^3$, $\quad S(r) = S(0) \exp\left(-b \overbrace{r^T D r}^{D(r) = \text{diffusivity}}\right)$

signal in absence of gradient pulse

constant related to exposure time

3x3 positive definite matrix

position vector

n-th order:

$$D(r) = \sum_{j_1=1}^{3} \sum_{j_2=1}^{3} \cdots \sum_{j_n=1}^{3} D_{j_1 j_2 \cdots j_n} r_{j_1} r_{j_2} \cdots r_{j_n}$$

1. Segmentation

Examples of well known techniques:

a) Active contour models

b) K-means

both are based on "good" similarity measures on the data

For DTI:

- Euclidean $\leftarrow$ standard on $\mathbb{R}^9$ = WRONG!
- log-Euclidean $\leftarrow$ $dist(p_1, p_2) = Tr(\log(p_1) - \log(p_2))^2$
- Spectral (spherical geometric) $\leftarrow$ decomp: $p = u \Lambda u^T$

$u \in SO(3)$, Λ diagonal

moreover, use the Hilbert anisotropy $\log \frac{\lambda_{max}}{\lambda_{min}} = H(p)$

localised active contour method:

based on "edge" functions, e.g.

$$g(u) = \frac{1}{1 + \|\nabla (G_\sigma * u)\|^p}$$

Gaussian density with variance σ

contour = zero level of ϕ

variants of Mumford–Shah funct.

$\Rightarrow$ gradient to move the function ϕ

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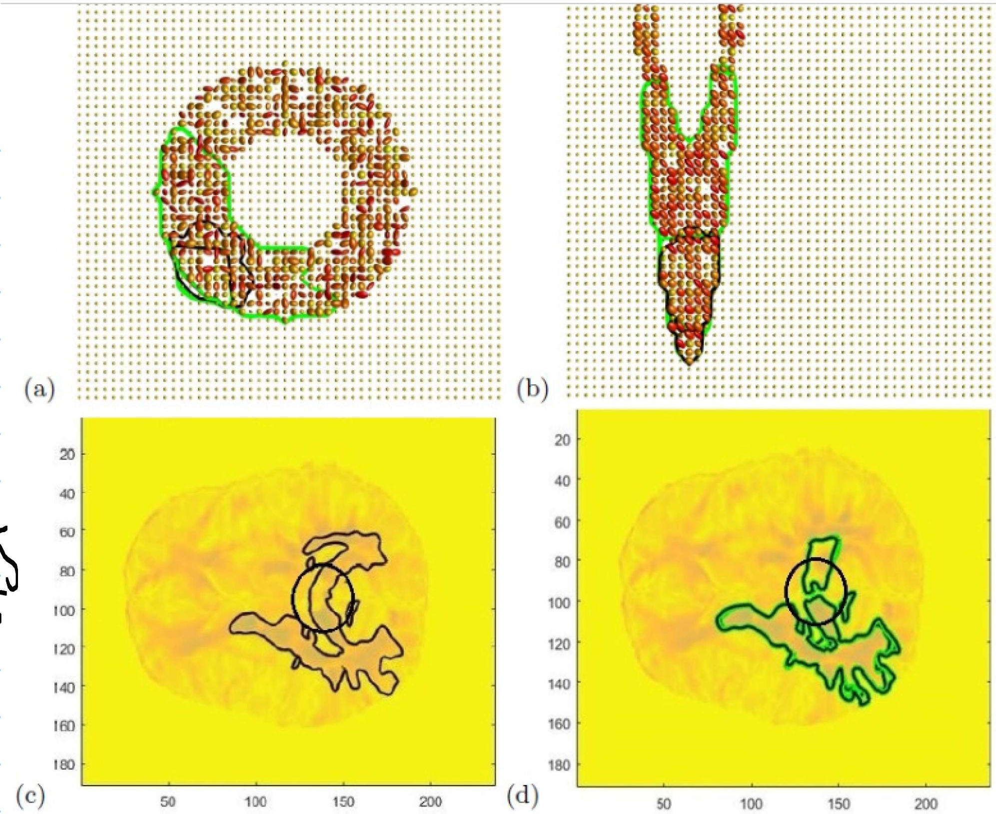
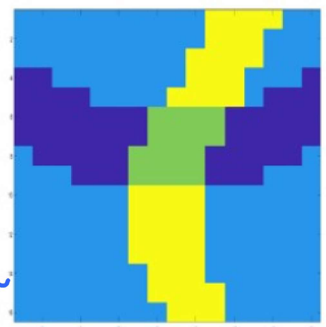


Fig. 2. In (a), slerp-SQ is faster (both curves shown at 521 iterative step, both are able to segment the object). In (b), LogE fails to evolve after 50th iteration, whereas slerp-SQ continued segmenting the whole object. Segmentation inside region of interest (roi) shown with ellipse, localization radius=20, z-slice=86, data size=191x236x171. In (c), LogE fails to deal with heterogeneity present in roi, while in (d), slerp-SQ is able to discern the heterogeneous data present within roi.

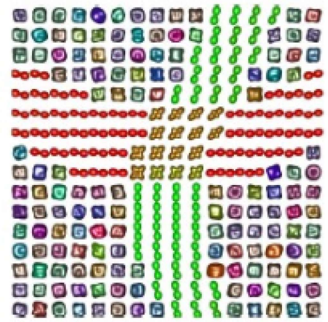
K-means (using HARDI - 4th order)

goal: distribute voxels into k classes

Step 1: exploit projections to 2nd order tensors



Step 2: exploit the non-linear dimensionality reduction (the graph-Laplacian obtained from the so called affinity matrix)



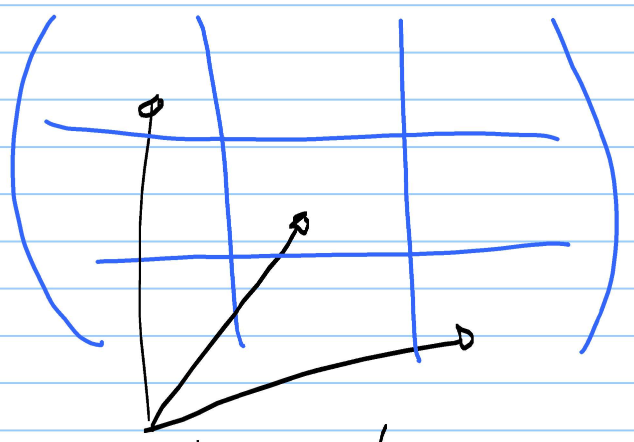
The image was created with additional random background noise (introducing randomness in partial fractions of the voxels, still nearly anisotropic), cf. [39]. The corresponding image without noise shows the same voxels in the fibers and uniform anisotropic background.

again the proper choice of metric, knowing their statistical properties, is crucial

D-projection:

4th order tensor is "flattened" as 9x9 matrix

$T =$



SPD's (under some cone)

$T : S^2\mathbb{R}^3 \to S^2\mathbb{R}^3$

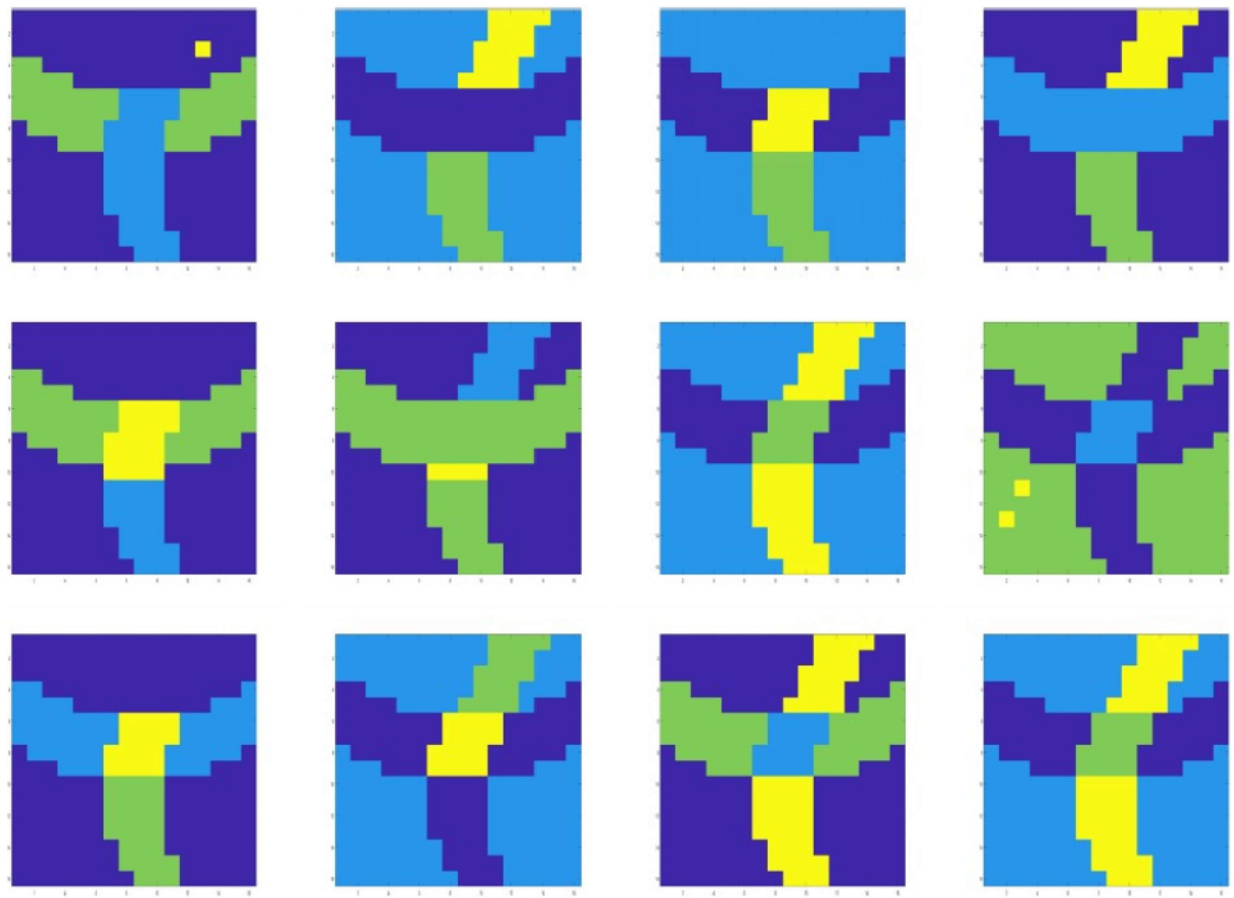


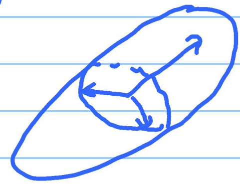
Fig. 3 Results with diverse metrics and projections. The rows show the LogE, SQ and slerpSQ metrics, from top to bottom, while the L projection appears in the first two columns, first without and then with the noise, and finally, the last two columns show the same for the D projection. The colors are chosen randomly (Color figure online)

2. Fiber tracking

<u>Goal</u>: find the fiber structure of the white matter!

<u>Idea</u>: use suitable Riemannian metric to get the fibers as geodesics.

<u>Difficulty</u>: crossing, merging and kissing fibers

naive DTI approach: $D \sim$  eigenvalues $\lambda_1, \lambda_2, \lambda_3$
principal axis = eigenvectors

D^t ... same eigenvectors,
eigenvalues $\lambda_1^t, \lambda_2^t, \lambda_3^t$

D^{-1} is the first choice...

enhancement: 1) use reasonable power, e.g. D^{-2}, D^{-4} $\Rightarrow$ "sharpening"

2) choose some better metric in the conformal class $\Rightarrow$ "β-scaled"

aim: make the cost very big in all directions in the isotropic regions

solution: use an "activation function" based on the Hilbert anisotropy

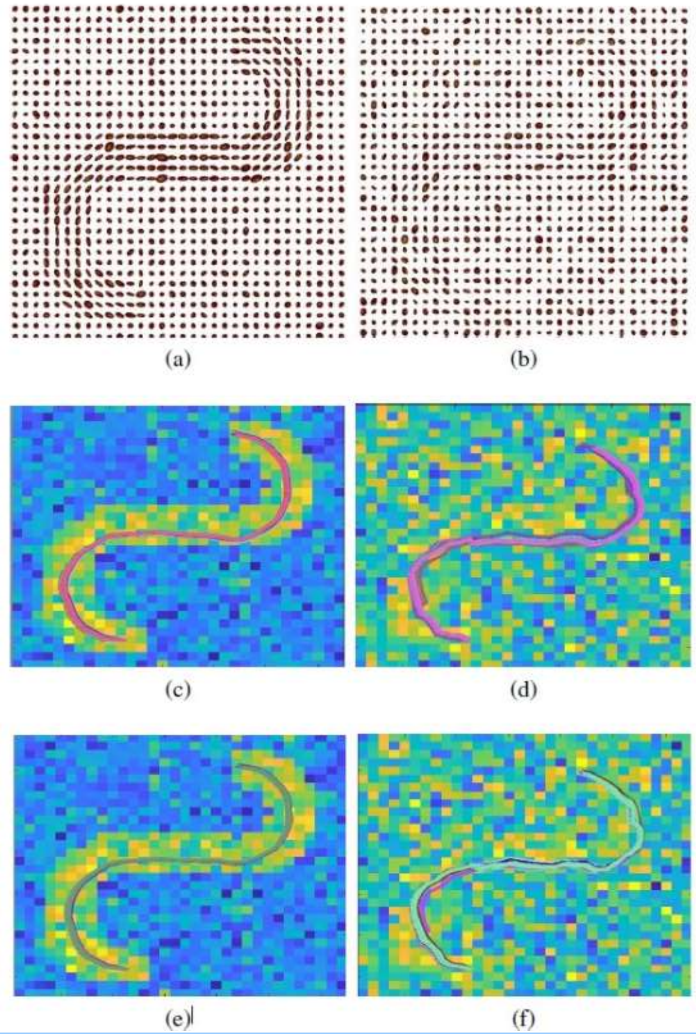


Fig. 11: (a) Reflected S-shaped fiber with Riccian noise 0.25, (b) Signal corrupted with Riccian noise = 0.30, (c) Ray-tracing with Principal eigenvector direction using adjugate and noise 0.25 (d) Ray-tracing with Principal eigenvector direction using adjugate and noise 0.30, (e) Ray-tracing with principal eigenvector direction using β-scaled metrics with $p = 2$ and Riccian noise 0.25, (f) Ray-tracing with principal eigenvector direction using β-scaled metrics with $p = 2$ and Riccian noise 0.30

I followed mainly these two papers:

BIHONEGN, Temesgen Tsegaye, Sumit KAUSHIK, Avinash BANSAL, Lubomír VOJTÍŠEK a Jan SLOVÁK. Geodesic fiber tracking in white matter using activation function. Computer Methods and Programs in Biomedicine. Elsevier, 2021, roč. 208, September, s. "106283", 14 s.

KAUSHIK, Sumit a Jan SLOVÁK. HARDI Segmentation via Fourth-Order Tensors and Anisotropy Preserving Similarity Measures. JOURNAL OF MATHEMATICAL IMAGING AND VISION. DORDRECHT: SPRINGER, 2019, roč. 61, č. 8, s. 1221-1234.

plus Sumit's PhD thesis