# Conformal transformations and the beginning of the Universe. Part III. 

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- A conformal transformation: $g \rightarrow \hat{g}=\frac{1}{\Omega^{2}} g$ results in the following transformation of the respective Ricci tensors:
$\hat{h}_{\mu v}=\Omega^{2} R_{\mu \nu}+2 \Omega \nabla \Omega_{\mu}+g_{\mu v}\left(\Omega-\Omega-3 g^{\prime}\left(\nabla \Omega,-\Omega^{\prime}\right)\right.$
- Contracting we get the following transformation of the Ricci scalars:

$$
\hat{n}=\Omega^{2} n+6 \Omega \square \Omega-12 g(\nabla \Omega, \nabla \Omega)
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- Interpreting $\hat{g}$ as the physical metric of spacetime, whose conformal compactified metric $g$ has $\mathscr{I}$ where $\Omega \rightarrow 0$, we see that the causal properties of are governed by the formula: $\hat{R}=-12 g(\nabla \Omega, \nabla \Omega)$.
- Recall that the signature is $(-\perp+\perp)$, so is spacelike if $\nabla \Omega$ is timelike, i.e. if the Ricci scalar $\hat{R}$ of the physical metric is positive, $R>0$.
- A conformal transformation: $g \rightarrow \hat{g}=\frac{1}{\Omega^{2}} g$ results in the following transformation of the respective Ricci tensors: $\hat{R}_{\mu \nu}=\Omega^{2} R_{\mu \nu}+2 \Omega \nabla_{\nu} \Omega_{\mu}+g_{\mu \nu}(\Omega \square \Omega-3 g(\nabla \Omega, \nabla \Omega))$
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- Using the Einstein equations satisfied by the metric $\hat{g}$,

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\hat{R}_{\mu \nu}-\frac{1}{2} \hat{R} \hat{g}_{\mu \nu}+\hat{\Lambda} \hat{g}_{\mu \nu}=\hat{T}_{\mu \nu}
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or

one can express this also in terms of the cosmological constant
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## Brief history of theoretical cosmology

- Perfect cosmological principle (Copernicus ?): The Universe is the same everywhere, in every direction, and at every moment of time.
- A General Relativity model of such Universe is due to Einstein (1917):

$$
M=\mathbb{R} \times \mathbb{S}^{3} \text {, and } g_{\text {Einst }}=-\mathrm{d} t^{2}+\Omega^{2} g_{\mathrm{B}},
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with $g_{s^{3}}$ is the standard metric on $\mathbb{S}^{3}$ of radius 1 , and $\Omega=$ const. This is the Einstein's static Universe model.

- The Ricci tensor for this spacetime is Ricci $=\frac{2}{n^{2}} g_{\mathbb{*}}$.This satisfies Einstein's equations

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- The models of the Universe compatible with this principle, due to Friedman, Lemaître, Robertson and Walker (FLRW) are such that the spacetime manifold $M$ is a bundle $S \rightarrow M \rightarrow \mathbb{R}$, with $S=\mathbb{H}^{3}$, or $\mathbb{R}^{3}$, or $\mathbb{S}^{3}$ - the spaces of constant curvature, with the spacetime metric $g=-\mathrm{d} t^{2}+\Omega^{2}(t) g_{s}$, in which $g_{s}$ is either the standard metric on the hyperbolic space $\mathbb{H}^{3}$, or the flat metric on $\mathbb{R}^{3}$, or the standard metric on the unit sphere $\mathbb{S}^{3}$.
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- $\Omega(t)=\alpha \cosh \left(\frac{t}{\alpha}\right)$ and $\varkappa=1$ : deSitter Universe; vacuum solution with $\Lambda=\frac{3}{a^{2}}$; spacetime of constant curvature.
- $\Omega(t)$ satisfies $\Omega\left(\Omega^{\prime 2}+x\right)=\frac{M}{4 \pi}+\frac{1}{3} \wedge \Omega^{3}$, with $M=$ const $>0$, $\Lambda=$ const; Friedman-Lemaître Universe with $\wedge$ of arbitrary sign, filled with dust of energy density $\mu=\frac{N}{\frac{2}{2} \pi n}$, comoving with 4 -velocity $u=-\mathrm{d} t$. For the Ricci scalar being positive one needs $\Lambda>-\mu / 4$.
- Note that all FLRW metrics are conformally flat!

Brief history of theoretical cosmology: simple FLRW models

- Universe spacetime: $S \rightarrow M \rightarrow \mathbb{R}$, with $g=-\mathrm{d} t^{2}+\Omega^{2}(t) g_{S}$.
- For example, there exists local coordinates $(x, y, z)$ on $S$ such that $g_{s}=\frac{\left.d x^{2}+d y^{2}+d z^{2}\right)}{\left(1+\frac{x}{4}\left(x^{2}+y^{2}-z^{2}\right)\right)^{2}}$, and $\varkappa=-1,0,1$ corresponds to $\mathbb{H}^{3}$, $\mathbb{R}^{3}$ and $\mathbb{S}^{3}$, respectively.
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## The scheme of Penrose's CCC is as follows: ${ }^{1}$

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> ${ }^{1}$ See: P. Tod (2015), 'The equations of Conformal Cyclic Cosmology', Gen. Rel. Grav. 47,https://doi.org/10.1007/s10714-01,5-1,859-7, for details.

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- The Universe consists of eons, each being a time oriented spacetime, whose conformal compactifications have spacelike $\mathscr{I}$. The Weyl tensor of the metric on each $\mathscr{I}$ is zero.
- Eons are ordered, and the conformal compactifications of consecutive eons, say $\# i$ and $\#(i+1)$, are glued together along $\mathscr{I}$ of the eon $\# i$, and $\mathscr{I}$ of the eon $\#(i+1)$.
- The vicinity of the matching surface (the wound) of eons \#i and $\#(i+1)$, this region Penrose calls bandaged region for the two eons, is equipped with the following three metrics, which are conformally flat at the wound:
- a I orentzian metric $a$ which is regular everywhere,
- a Lorentzian metric $̆$ g which represents the physical metric of the eon \#(i+1), and which is singular at the wound,
- a Lorentzian metric $\hat{g}$, which represents the physical metric of the eon \#i, and which expands at the wound.
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- The Universe consists of eons, each being a time oriented spacetime, whose conformal compactifications have spacelike $\mathscr{I}$. The Weyl tensor of the metric on each $\mathscr{I}$ is zero.
- Eons are ordered, and the conformal compactifications of consecutive eons, say $\# i$ and $\#(i+1)$, are glued together along $\mathscr{I}^{+}$of the eon $\# i$, and $\mathscr{I}^{-}$of the eon $\#(i+1)$.
- The vicinity of the matching surface (the wound) of eons $\# i$ and \#(i+1), this region Penrose calls bandaged region for the two eons, is equipped with the following three metrics, which are conformally flat at the wound:
- a Lorentzian metric $g$ which is regular everywhere,
- a Lorentzian metric ğ, which represents the physical metric of the eon $\#(i+1)$, and which is singular at the wound,
- a Lorentzian metric $\hat{g}$, which represents the physical metric of the eon $\# i$, and which expands at the wound.
- In any bandaged region, the three metrics $g, g$ and $\hat{g}$, are conformally related.
- How to make this relation specific is debatable, but Penrose proposes that
$\check{g}=\Omega^{2} g$, and $\hat{g}=\frac{1}{\Omega^{2}} g$, with $\Omega \rightarrow 0$ on the wound.
- The metric $g$ in eon $\#(i+1)$ is a physical metric there. Likewise, the metric $\hat{g}$ in eon $\# i$ is a physical metric there.
- Of course, the metric $g$ g in eon \#(i+1), and the metric $\hat{g}$ in eon \#i, as physical spacetime metrics, should satisfy Einstein's equations in \#(i+1) and \#i, respectively.
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Penrose's Conformal yclic osmology


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## Modelling Penrose's CCC scenario

- Question: How to make a model of Penrose's bandaged region?
- One needs a function $\Omega$, vanishing on some spacelike hypersurface, such that if $g=\Omega^{2} g$ satisfies Einstein equations with some physically reasonable energy momentum tensor, then $g=\Omega$ also satisfies Einstein equations with possibly different, but still physically reasonable energy momentum tensor.
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## Polytrope perfect fluids in FLRW models

- From now on I restrict myself to FLRW metrics with $\kappa=1$, $g=-\mathrm{d} t^{2}+\Omega^{2}(t) r_{0}^{2}\left(\mathrm{~d} \chi^{2}+\sin ^{2} \chi\left(\mathrm{~d} \theta^{2}+\sin ^{2} \theta \mathrm{~d} \phi^{2}\right)\right)$.
- It is convenient to introduce a conformal time $n=\int \frac{\mathrm{d} t}{d(t)}$ so that the FLRW metric looks

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g=\Omega^{2}(\eta)\left(-\mathrm{d} \eta^{2}+r_{0}^{2}\left(\mathrm{~d} \chi^{2}+\sin ^{2} \chi\left(\mathrm{~d} \theta^{2}+\sin ^{2} \theta \mathrm{~d} \phi^{2}\right)\right)\right),
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i.e. $g=\Omega^{2}(\eta) g_{\text {Einst }}$.

- This parametrization is very convenient since taking $u=-\Omega(\eta) \mathrm{d} \eta$, the most general FLRW metric $g$ satisfying Einstein's equations

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## Symmetry of solutions conformal to the Einstein Universe

## Theorem

If $\Omega=\Omega(\eta)$ is such that $g ̆=\Omega^{2} g_{\text {Einst }}$ satisfies Einstein's equations, with $\Lambda=0$, and with the energy momentum tensor $\bar{T}$ of a perfect fluid, whose presure $\check{p}$ is proportional to the energy density $\check{\mu}$, via $\check{p}=\check{w} \check{\mu}, \check{w}=$ const, then
$\hat{g}=\frac{1}{\Omega^{2}}$ gEinst satisfies Einstein's equations, with $\Lambda=0$, and with the energy momentum tensor $\hat{T}$ of a perfect fluid, whose presure $\hat{p}$ and the energy density $\hat{\mu}$ are related by $\hat{p}=\hat{w} \hat{\mu}$ with


The Ricci sclar of the metric $g$ is

so it is positive if $-1 \leq \check{w}<1 / 3$ (recall the energy conditions

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If $\Omega=\Omega(\eta)$ is such that $g ̆=\Omega^{2} g_{\text {Einst }}$ satisfies Einstein's equations, with $\Lambda=0$, and with the energy momentum tensor $\check{T}$ of a perfect fluid, whose presure $\check{p}$ is proportional to the energy density $\check{\mu}$, via $\check{p}=\check{w} \check{\mu}, \check{w}=$ const, then
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\hat{g}=\frac{1}{\Omega^{2}} g_{\text {Einst }}
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\hat{w}=-\frac{1}{3}(2+3 \check{w})
$$

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$\square$

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$$
R=\frac{3(1-3 \check{W})}{\Omega_{0}^{2} r_{0}^{2}\left(\sin ^{6} \frac{(1+3 \check{W}) \eta}{2 r_{0}}\right)^{\frac{1+w}{1+3 \check{w}}}} \text { if } \check{W} \neq-1 / 3 \text { and }
$$

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$R=\frac{3(1-3 \check{w})}{\Omega_{0}^{2} r_{0}^{2}\left(\sin ^{6} \frac{(1+3 \check{w}) \eta}{2 r_{0}}\right)^{\frac{1+w}{1+3 w}}}$ if $\check{w} \neq-1 / 3$ and $R=\frac{6\left(1+b^{2} r_{0}^{2}\right)}{\Omega_{0}^{2} r_{0}^{2} \exp (2 b \eta)}$ if $\check{w}=-1 / 3$, so it is positive if $-1 \leq \check{W}<1 / 3$ (recall the energy conditions
$-1 \leq \check{w} \leq 1)$.




$$
\begin{aligned}
& \text { 1-radiation } \\
& (-1) \leftrightarrow \frac{1}{3} \\
& \text { matter - gas of } \\
& 0 \longmapsto-2 / 3 \\
& \text { less of strings. } \\
& -1 / 6 \text { s-7 }-1 / 3
\end{aligned}
$$

Suspiscious points: $\check{w}=-1,1 / 3$ (cosmological constant radiation), since the scalar curvature $R=0$, when $\check{w}=1 / 3$; and $\mathscr{w}=-1 / 3$ (gas of strings), when $\Omega \neq 0$ on $\mathscr{I}$.


$$
\begin{array}{r}
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- We come back to the FLRW metric $\check{g}=-\mathrm{d} t^{2}+\Omega^{2}(t) r_{0}^{2} g_{s^{3}}$.
- We write it as $\check{g}=\Omega^{2}(t)\left(-\frac{\mathrm{d}^{2}}{\Omega^{2}(t)}+r_{0}^{2} g_{s^{3}}\right)$, so that it is clear that $\breve{g}=\Omega^{2}(t) g_{\text {Einst }}$.
- Then the condition that g satisfies perfect fluid Eisntein's equations with $\check{u}=-\mathrm{d} t, \check{p}=\check{w} \check{\mu}$, and the cosmological constant $\Lambda$, is equivalent to the following ODE for $\Omega$ :
$2 t^{2} \Omega \Omega^{\prime \prime}-\left(1+3 W_{1}\right)\left(1+t^{-2} \Omega^{\prime 2}\right)+\left(1+\psi_{1} x+2 \Omega^{2}\right.$.
- We want that $\check{W}=$ const and that $\hat{g}=\frac{1}{\Omega^{2}} g_{\text {Einst }}$ satisfies perfect fluid Eisntein's equations with $\hat{u}=-\frac{\mathrm{dt}}{\Omega^{2}}, \hat{p}=\hat{w} \hat{\mu}$, the cosmological constant $\hat{\wedge}$, and $\hat{W}=$ const.
- From the Einstein's equations for $\hat{g}$ we easilly calculate $\hat{w}$, and forcing it to be constant, because of the above ODE satisfied by $\Omega$, we find that it is possible provided that:

$$
\check{X} \hat{\Lambda}(1+\mathscr{W})(1-3 \mathscr{W})=0 .
$$

- Thus, a neccessary condition for both $\Omega$ and $\Omega^{-1}$ to describe the polytropes, is that either one of the $\wedge$ s is zero, or $\check{w}$ is of the 'radiation- $\wedge$ ' type.
- We come back to the FLRW metric $\check{g}=-\mathrm{d} t^{2}+\Omega^{2}(t) r_{0}^{2} g_{\mathbb{S}^{3}}$.
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- Then the condition that $g$ satisfies perfect fluid Eisntein's equations with $\check{u}=-\mathrm{d} t, \check{p}=\check{w} \check{\mu}$, and the cosmological constant $x$, is equivalent to the following ODE for $\Omega$ :
- We want that $\check{w}=$ const and that $\hat{g}=\frac{1}{\Omega^{2}} g_{\text {Einst }}$ satisfies perfect fluid Eisntein's equations with $\hat{u}=-\frac{d t}{\Omega^{2}}, \hat{p}=\hat{w} \hat{\mu}$, the cosmological constant $\hat{\wedge}$, and $\hat{W}=$ const.
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$$
2 r_{0}^{2} \Omega \Omega^{\prime \prime}=-(1+3 \check{W})\left(1+r_{0}^{2} \Omega^{\prime 2}\right)+(1+\check{w}) \check{\Lambda} r_{0}^{2} \Omega^{2} .
$$

- We want that $\check{W}=$ const and that $\hat{g}=\frac{1}{\Omega^{2}} g_{\text {Einst }}$ satisfies perfect
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the
cosmological constant $\hat{\Lambda}$, and $\hat{w}$
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$$

- We want that $\check{W}=$ const and that $\hat{g}=\frac{1}{\Omega^{2}} g_{\text {Einst }}$ satisfies perfect fluid Eisntein's equations with $\hat{u}=-\frac{d t}{\Omega^{2}}$,
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$$
2 r_{0}^{2} \Omega \Omega^{\prime \prime}=-(1+3 \check{W})\left(1+r_{0}^{2} \Omega^{\prime 2}\right)+(1+\check{w}) \check{\wedge} r_{0}^{2} \Omega^{2} .
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and $\Omega^{-1}$ to describe the polytropes, is that either one of the $\wedge$ s is zero, or $w$ is of the 'radiation-- 'type.
- We come back to the FLRW metric $\check{g}=-\mathrm{d} t^{2}+\Omega^{2}(t) r_{0}^{2} g_{\mathbb{S}^{3}}$.
- We write it as $\check{g}=\Omega^{2}(t)\left(-\frac{\mathrm{dt}}{\Omega^{2}}(t)+r_{0}^{2} g_{\mathbb{S}^{3}}\right)$, so that it is clear that $\check{g}=\Omega^{2}(t) g_{\text {Einst }}$.
- Then the condition that $\check{g}$ satisfies perfect fluid Eisntein's equations with $\check{u}=-\mathrm{d} t, \check{p}=\check{w} \check{\mu}$, and the cosmological constant $\Lambda$, is equivalent to the following ODE for $\Omega$ :
$2 r_{0}^{2} \Omega \Omega^{\prime \prime}=-(1+3 \check{W})\left(1+r_{0}^{2} \Omega^{\prime 2}\right)+(1+\check{w}) \check{\Lambda} r_{0}^{2} \Omega^{2}$.
- We want that $\check{w}=$ const and that $\hat{g}=\frac{1}{\Omega^{2}} g_{\text {Einst }}$ satisfies perfect fluid Eisntein's equations with $\hat{u}=-\frac{\mathrm{d} t}{\Omega^{2}}, \hat{p}=\hat{w} \hat{\mu}$, the cosmological constant $\hat{\Lambda}$, and $\hat{w}=$ const.
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## Possible generalizations

- Consider (special) Bianchi models: $\check{g}=-\mathrm{d} t^{2}+\Omega^{2}(t) h_{i j} \sigma^{i} \sigma^{j}$, where $\sigma^{i}, i, j=1,2,3$, are left invariant forms on a 3-dimensiona Lie group $G$, and (h) is a symmetric positive definite matrix. Here the Universe manifold is $M=\mathbb{R} \times G$, and the time variable $t$ is along the $\mathbb{R}$ factor. For each Bianchi type of $G$, decide which metric should play the role of $g_{\text {Einst }}$. In other words: find a preferred basis of the left invariant forms on $G$, so that the counterpart of $g_{\text {Einst }}$ is $g_{E}=-\Omega^{-2}(t) d t^{2}+\left(\sigma^{1}\right)^{2}+\left(\sigma^{2}\right)^{2}+\left(\sigma^{3}\right)^{2}$; and then play the game with $\check{g}=\Omega^{2} g_{E}$ and $\hat{g}=\Omega^{-2} g_{E}$, similar to this I was describing in this talk. My game was Bianchi IX, i.e. I took $G=S U(Z)=\sim^{2}$.
- More generally, take as $g_{E}$ the metric $g_{E}=-\Omega^{-2}(t) \mathrm{d} t^{2}+g_{S}$, where $M=R \times S$, and $\left(S, g_{s}\right)$ is a 3D Riemannian manifold (possibly satisiying some interesting equations, or as in the previous case symmetry conditions). Again play the game with polytropic perfect fluids for $\check{g}=\Omega^{2} g_{E}$ and $\hat{g}=\Omega^{-2} g_{E}$
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