Conformal transformations and the beginning of the Universe. Part III.

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Conformal transformations of spacetimes and Ricci

- A conformal transformation: $g \to \hat{g} = \frac{1}{\Omega^2} g$ results in the following transformation of the respective Ricci tensors:
$$\hat{R}_{\mu\nu} = \Omega^2 R_{\mu\nu} + 2\Omega \nabla_\nu \Omega_\mu + g_{\mu\nu} \left(\Omega \Box \Omega - 3g(\nabla\Omega, \nabla\Omega) \right)$$
- Contracting we get the following transformation of the Ricci scalars:
$$\hat{R} = \Omega^2 R + 6\Omega \Box \Omega - 12g(\nabla\Omega, \nabla\Omega).$$
- Interpreting $\hat{g}$ as **the physical metric of spacetime**, whose conformal compactified metric g has $\mathscr{I}$ where $\Omega \to 0$, we see that the **causal properties of** $\mathscr{I}$ are governed by the formula:
$$\hat{R} = -12g(\nabla\Omega, \nabla\Omega).$$
- Recall that the signature is $(-, +, +, +)$, so $\mathscr{I}$ **is spacelike** if $\nabla\Omega$ is timelike, i.e. **if the Ricci scalar** $\hat{R}$ of the physical metric is **positive**, $\hat{R} > 0$.

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or
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one can express this also in terms of the cosmological constant $\hat{\Lambda}$ of the physical spacetime and the trace of its energy momentum tensor $\hat{T}$:
$$4\hat{\Lambda} - \hat{T} > 0.$$
- This, in particular means that if close to $\mathscr{I}$ the trace of the energy momentum $\hat{T}$ vanishes, a **positive cosmological constant $\hat{\Lambda}$ makes $\mathscr{I}$ spacelike.**

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- In particular the Ricci scalar $R = \frac{6}{\Omega^2}$ **is positive.** This means that Einstein's model has **spacelike** $\mathscr{I}$.

- **Perfect cosmological principle** (Copernicus ?): The Universe is the same **everywhere**, **in every direction**, and **at every moment of time.**
- A **General Relativity model** of such Universe is **due to Einstein** (1917):
$$M = \mathbb{R} \times \mathbb{S}^3, \text{ and } g_{Einst} = -\mathrm{d}t^2 + \Omega^2 g_{\mathbb{S}^3},$$
with $g_{\mathbb{S}^3}$ is the standard metric on $\mathbb{S}^3$ of radius 1, and $\Omega = const$. This is the **Einstein's static Universe model.**
- The Ricci tensor for this spacetime is $Ricci = \frac{2}{\Omega^2} g_{\mathbb{S}^3}$.This satisfies Einstein's equations
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- **Note** that **all** FLRW metrics **are conformally flat!**

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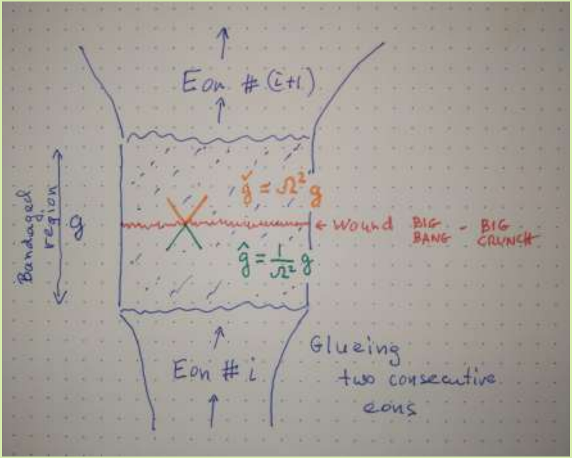
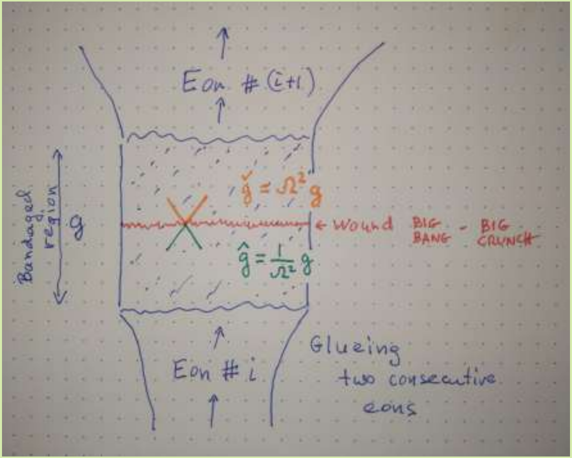

Eon # (i+1)
Bandaged region
g
$\check{g} = \Omega^2 g$
← Wound
BIG BANG - BIG CRUNCH
$\hat{g} = \frac{1}{\Omega^2} g$
Glueing two consecutive eons
Eon # i


Eon # (i+1)
$\check{g} = \Omega^2 g$
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Polytrope perfect fluids in FLRW models

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$$g = -\mathrm{d}t^2 + \Omega^2(t) r_0^2 \left(\mathrm{d}\chi^2 + \sin^2\chi (\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \right).$$
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$$\Omega(\eta) = \Omega_0 \left(\sin^2 \frac{(1+3w)\eta}{2r_0} \right)^{\frac{1}{1+3w}} \text{ if } w \neq -\frac{1}{3},$$
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so it is **positive** if $\boxed{-1 \leq \check{w} < 1/3}$ (recall the energy conditions $-1 \leq \check{w} \leq 1$).

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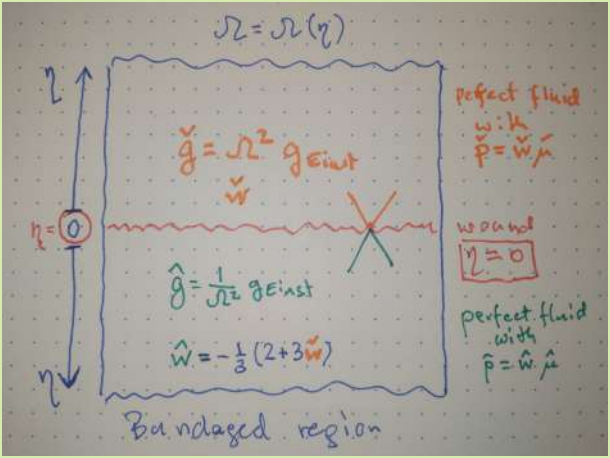
Transformation $\breve{g} \to \hat{g} \to \breve{g} \to \hat{g} \to \ldots$ of fluids



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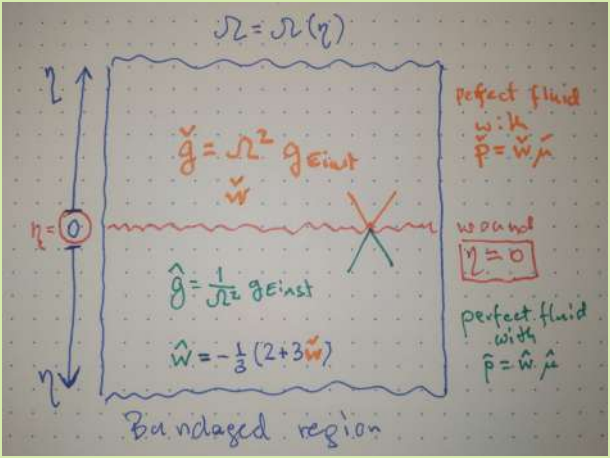

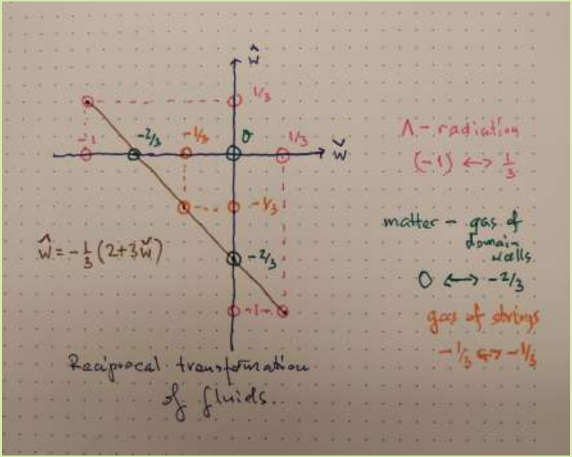

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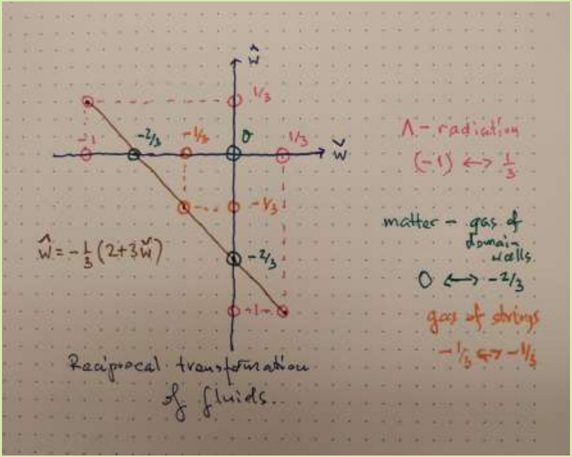

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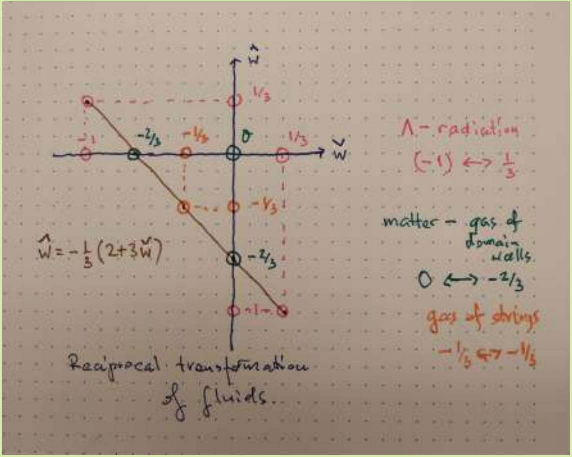

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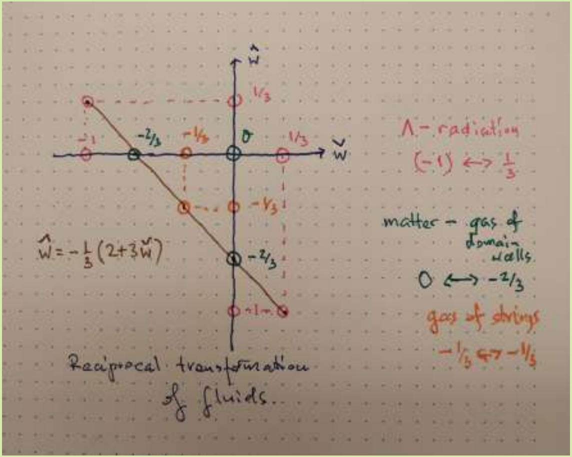

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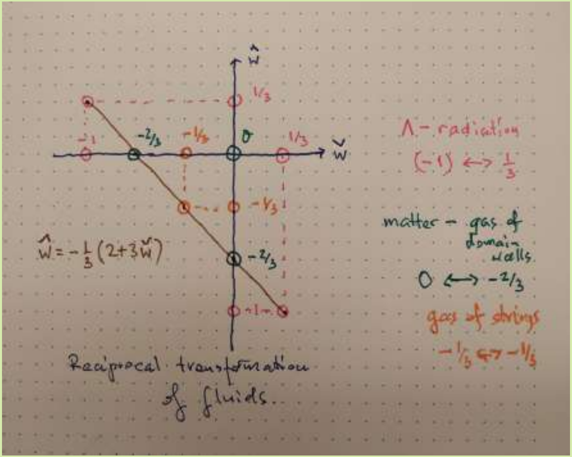

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Transformation $\check{g} \to \hat{g} \to \check{g} \to \hat{g} \to \dots$ of fluids: more careful approach

- We come back to the FLRW metric $\check{g} = -\mathrm{d}t^2 + \Omega^2(t) r_0^2 g_{\mathbb{S}^3}$.
- We write it as $\check{g} = \Omega^2(t)\big(-\frac{\mathrm{d}t^2}{\Omega^2(t)} + r_0^2 g_{\mathbb{S}^3}\big)$, so that it is clear that $\check{g} = \Omega^2(t) g_{Einst}$.
- Then the condition that $\check{g}$ satisfies perfect fluid Eisntein's equations with $\check{u} = -\mathrm{d}t$, $\check{p} = \check{w}\check{\mu}$, and the cosmological constant $\check{\Lambda}$, is equivalent to the following ODE for Ω:

$$2r_0^2 \Omega\Omega'' = -(1+3\check{w})(1 + r_0^2 \Omega'^2) + (1+\check{w})\check{\Lambda} r_0^2 \Omega^2.$$

- We want that $\check{w} = const$ and that $\hat{g} = \frac{1}{\Omega^2} g_{Einst}$ satisfies perfect fluid Eisntein's equations with $\hat{u} = -\frac{\mathrm{d}t}{\Omega^2}$, $\hat{p} = \hat{w}\hat{\mu}$, the cosmological constant $\hat{\Lambda}$, and $\hat{w} = const$.
- From the Einstein's equations for $\hat{g}$ we easilly calculate $\hat{w}$, and forcing it to be constant, because of the above ODE satisfied by Ω, we find that it is possible provided that:

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THANK YOU!