Conformal transformations and the beginning of the Universe. Part I.

Pawel Nurowski

Centrum Fizyki Teoretycznej
Polska Akademia Nauk

GRIEG running seminar nr. 2, 1.12.2020

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors u always satisfy $g(u,u) < 0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
  - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u) = 0$;

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors u always satisfy $g(u,u) < 0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
  - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u) = 0$;

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors u always satisfy $g(u,u) < 0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
  - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u) = 0$;

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors u always satisfy $g(u,u) < 0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
  - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u) = 0$;

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors u always satisfy $g(u,u) < 0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
  - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u) = 0$;

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors u always satisfy $g(u,u)<0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u)>0$,
  - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u)=0$;

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors u always satisfy $g(u,u) < 0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
  - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u) = 0$;

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors u always satisfy $g(u,u) < 0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
  - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u) = 0$;

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
    - **timelike** curves: whose tangent vectors u always satisfy $g(u,u) < 0$,
    - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
    - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u) = 0$;

Quick intro to **General Relativity** theory:

- the arena for all physical events is a **spacetime** – a FOUR-dimensional manifold M equipped with a *metric* g of *Lorentzian signature* $(-,+,+,+)$,
- points of M – are **physical events**; curves in M – are histories of events,
- because of the Lorentzian signature, there are **three categories of curves**:
  - **timelike** curves: whose tangent vectors u always satisfy $g(u,u) < 0$,
  - **spacelike** curves: whose tangent vectors satisfy $g(u,u) > 0$,
  - **null**, or using Elie Cartan's name, **optical** curves: whose tangent *nonzero* vectors satisfy $g(u,u) = 0$;

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons∼ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy
$$\nabla_u u = 0;$$
the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons∼ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy

$$\nabla_u u = 0;$$

the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons~ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy

$$\nabla_u u = 0;$$

the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons~particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy

$$\nabla_u u = 0;$$

the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons $\sim$ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy

$$\nabla_u u = 0;$$

the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons$\sim$ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy

$$\nabla_u u = 0;$$

the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons $\sim$ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy

$$\nabla_u u = 0;$$

the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons~ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy

$$\nabla_u u = 0;$$

the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons ∼ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy

$$\nabla_u u = 0;$$

the word **normalized** means that $g(u, u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- **Curves** representing movement of particles in spacetime **are** particles' **worldlines**; physically **realistic particles** have worldlines which are:
  - either everywhere **timelike**, if they have mass, or
  - **optical**, if they are massless (they represent e.g. photons$\sim$ particles of light);
  - curves whose tangent vectors are **never spacelike** are called **causal**; causal curves correspond to **worldlines of physically acceptable particles**;
- Paricles in **free fall** have worldlines, which are **affinely** parametrized **causal geodesics.** Their **normalized** tangent vectors u satisfy
$$\nabla_u u = 0;$$
the word **normalized** means that $g(u,u) = -1$ (for particles with mass) or 0 (for masless particles as e.g. photons).

- The movement of test particles in free fall in gravitational field is determined by the Levi-Civita connection ∇ of the metric g, and the Newtonian **gravitational force** is incorporated in the notion of this connection,
- In GR every spacetime satisfies **Einstein's field equations**
$$Ric - \frac{1}{2}Rg + \Lambda g = \kappa T,$$
where Λ is a (cosmological) constant, κ is a universal constant (we choose units that it is equal to 1), Ric is the Ricci tensor of g, R is its Ricci scalar, and T is the **energy momentum tensor**, which represents the matter content of spacetime;
- Once g satisfying Einstein's equations is given in M, the dynamics of **free** particles's movement is goverened by a simple rule: knowing a position p and velocity u of a particle at p, follow a geodesic passing through p and tangent to u; this is the worldline of the considered particle.

- The movement of test particles in free fall in gravitational field is determined by the Levi-Civita connection ∇ of the metric g, and the Newtonian **gravitational force** is incorporated in the notion of this connection,
- In GR every spacetime satisfies **Einstein's field equations**

$$Ric - \frac{1}{2}Rg + \Lambda g = \kappa T,$$

where Λ is a (cosmological) constant, κ is a universal constant (we choose units that it is equal to 1), Ric is the Ricci tensor of g, R is its Ricci scalar, and T is the **energy momentum tensor**, which represents the matter content of spacetime;
- Once g satisfying Einstein's equations is given in M, the dynamics of **free** particles's movement is goverened by a simple rule: knowing a position p and velocity u of a particle at p, follow a geodesic passing through p and tangent to u; this is the worldline of the considered particle.

- The movement of test particles in free fall in gravitational field is determined by the Levi-Civita connection ∇ of the metric g, and the Newtonian **gravitational force** is incorporated in the notion of this connection,
- In GR every spacetime satisfies **Einstein's field equations**

$$Ric - \frac{1}{2}Rg + \Lambda g = \kappa T,$$

where Λ is a (cosmological) constant, κ is a universal constant (we choose units that it is equal to 1), Ric is the Ricci tensor of g, R is its Ricci scalar, and T is the **energy momentum tensor**, which represents the matter content of spacetime;
- Once g satisfying Einstein's equations is given in M, the dynamics of **free** particles's movement is goverened by a simple rule: knowing a position p and velocity u of a particle at p, follow a geodesic passing through p and tangent to u; this is the worldline of the considered particle.

- The movement of test particles in free fall in gravitational field is determined by the Levi-Civita connection ∇ of the metric g, and the Newtonian **gravitational force** is incorporated in the notion of this connection,
- In GR every spacetime satisfies **Einstein's field equations**
$$Ric - \frac{1}{2}Rg + \Lambda g = \kappa T,$$
where Λ is a (cosmological) constant, κ is a universal constant (we choose units that it is equal to 1), Ric is the Ricci tensor of g, R is its Ricci scalar, and T is the **energy momentum tensor**, which represents the matter content of spacetime;
- Once g satisfying Einstein's equations is given in M, the dynamics of **free** particles's movement is goverened by a simple rule: knowing a position p and velocity u of a particle at p, follow a geodesic passing through p and tangent to u; this is the worldline of the considered particle.

- The movement of test particles in free fall in gravitational field is determined by the Levi-Civita connection ∇ of the metric g, and the Newtonian **gravitational force** is incorporated in the notion of this connection,
- In GR every spacetime satisfies **Einstein's field equations**
$$Ric - \frac{1}{2} Rg + \Lambda g = \kappa T,$$
where Λ is a (cosmological) constant, κ is a universal constant (we choose units that it is equal to 1), Ric is the Ricci tensor of g, R is its Ricci scalar, and T is the **energy momentum tensor**, which represents the matter content of spacetime;
- Once g satisfying Einstein's equations is given in M, the dynamics of **free** particles's movement is govered by a simple rule: knowing a position p and velocity u of a particle at p, follow a geodesic passing through p and tangent to u; this is the worldline of the considered particle.

- The movement of test particles in free fall in gravitational field is determined by the Levi-Civita connection ∇ of the metric g, and the Newtonian **gravitational force** is incorporated in the notion of this connection,
- In GR every spacetime satisfies **Einstein's field equations**
$$Ric - \frac{1}{2}Rg + \Lambda g = \kappa T,$$
where Λ is a (cosmological) constant, κ is a universal constant (we choose units that it is equal to 1), Ric is the Ricci tensor of g, R is its Ricci scalar, and T is the **energy momentum tensor**, which represents the matter content of spacetime;
- Once g satisfying Einstein's equations is given in M, the dynamics of **free** particles's movement is goverened by a simple rule: knowing a position p and velocity u of a particle at p, follow a geodesic passing through p and tangent to u; this is the worldline of the considered particle.

- The movement of test particles in free fall in gravitational field is determined by the Levi-Civita connection ∇ of the metric g, and the Newtonian **gravitational force** is incorporated in the notion of this connection,
- In GR every spacetime satisfies **Einstein's field equations**
$$Ric - \frac{1}{2}Rg + \Lambda g = \kappa T,$$
where Λ is a (cosmological) constant, κ is a universal constant (we choose units that it is equal to 1), Ric is the Ricci tensor of g, R is its Ricci scalar, and T is the **energy momentum tensor**, which represents the matter content of spacetime;
- Once g satisfying Einstein's equations is given in M, the dynamics of **free** particles's movement is goverened by a simple rule: knowing a position p and velocity u of a particle at p, follow a geodesic passing through p and tangent to u; this is the worldline of the considered particle.

Energy momentum tensors

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho} = \delta^{\mu}{}_{\rho}$.
- Here $T_{\mu\nu} = T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^{\mu}{}_{\nu} = g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^{\mu}{}_{\nu} = R^{\mu}{}_{\nu} + (\Lambda - \frac{1}{2}R)\delta^{\mu}{}_{\nu}$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T = \lambda_R + \Lambda - \frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho} = \delta^{\mu}{}_{\rho}$.
- Here $T_{\mu\nu} = T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^{\mu}{}_{\nu} = g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^{\mu}{}_{\nu} = R^{\mu}{}_{\nu} + (\Lambda - \frac{1}{2}R)\delta^{\mu}{}_{\nu}$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T = \lambda_R + \Lambda - \frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho} = \delta^{\mu}{}_{\rho}$.
- Here $T_{\mu\nu} = T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^{\mu}{}_{\nu} = g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^{\mu}{}_{\nu} = R^{\mu}{}_{\nu} + (\Lambda - \frac{1}{2}R)\delta^{\mu}{}_{\nu}$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T = \lambda_R + \Lambda - \frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho} = \delta^{\mu}{}_{\rho}$.
- Here $T_{\mu\nu} = T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^{\mu}{}_{\nu} = g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^{\mu}{}_{\nu} = R^{\mu}{}_{\nu} + (\Lambda - \frac{1}{2}R)\delta^{\mu}{}_{\nu}$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T = \lambda_R + \Lambda - \frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho} = \delta^{\mu}{}_{\rho}$.
- Here $T_{\mu\nu} = T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^{\mu}{}_{\nu} = g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^{\mu}{}_{\nu} = R^{\mu}{}_{\nu} + (\Lambda - \frac{1}{2}R)\delta^{\mu}{}_{\nu}$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T = \lambda_R + \Lambda - \frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho} = \delta^{\mu}{}_{\rho}$.
- Here $T_{\mu\nu} = T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^{\mu}{}_{\nu} = g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^{\mu}{}_{\nu} = R^{\mu}{}_{\nu} + (\Lambda - \frac{1}{2}R)\delta^{\mu}{}_{\nu}$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T = \lambda_R + \Lambda - \frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho} = \delta^\mu{}_\rho$.
- Here $T_{\mu\nu} = T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^\mu{}_\nu = g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^\mu{}_\nu = R^\mu{}_\nu + (\Lambda - \frac{1}{2}R)\delta^\mu{}_\nu$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T = \lambda_R + \Lambda - \frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

- Einstein's equations: $R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+\Lambda g_{\mu\nu}=T_{\mu\nu}$, $R=R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho}=\delta^\mu{}_\rho$.
- Here $T_{\mu\nu}=T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^\mu{}_\nu=g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^\mu{}_\nu=R^\mu{}_\nu+(\Lambda-\frac{1}{2}R)\delta^\mu{}_\nu$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T=\lambda_R+\Lambda-\frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho} = \delta^{\mu}{}_{\rho}$.
- Here $T_{\mu\nu} = T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^{\mu}{}_{\nu} = g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^{\mu}{}_{\nu} = R^{\mu}{}_{\nu} + (\Lambda - \frac{1}{2}R)\delta^{\mu}{}_{\nu}$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T = \lambda_R + \Lambda - \frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, and $g^{\mu\nu}g_{\nu\rho} = \delta^\mu{}_\rho$.
- Here $T_{\mu\nu} = T_{\nu\mu}$ is *symmetric*. Although $T_{\mu\nu}$ is symmetric, the *endomorphism* tensor $T^\mu{}_\nu = g^{\mu\rho}T_{\rho\nu}$, due to the Lorentzian signature of the metric g, is *not*. Many algebraic types of T!
- Since $T^\mu{}_\nu = R^\mu{}_\nu + (\Lambda - \frac{1}{2}R)\delta^\mu{}_\nu$ the eigenvalues λ_T of the endomorphism T **differ merely by a shift** $\lambda_T = \lambda_R + \Lambda - \frac{1}{2}R$ from the eigenvalues λ_R of Ric. Therefore, due to the Einstein's equations we may speak about algebraic classification of the energy momentum tensor, or Ricci tensor, and use for it the usual Jordan classification of endomorphisms in $\mathbb{R}^4$.
- Note however, that because of signature of the metric g one can atribute a **causality** to the eigenspaces, telling if the eigenspace is **timelike**, **spacelike** or **optical** (Plebański!).

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four.** We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant**.

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** $(\Lambda \neq 0)$ or **without** $(\Lambda = 0)$ **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy aspect of **classical matter and/or fields** is described in terms of the symmetric energy momentum tensor coming from an appropriate physical theory. Some of these tensors are purely phenomelogical (perfect fluid), some of them are deeper consequences of a theory (e.g. electromagnetic field). I review few of them now.

- The simplest energy momentum tensor, is the energy momentum tensor of **cosmological constant type**: the Ricci tensor has **one real eigenvalue** $\Lambda = \frac{1}{4}R$ **of multiplicity four**. We have $Ric = \Lambda g$. The metric g satisfies **mathematicans' Einstein's equations.**
- A special case is if the **quadruple eigenvalue is equal to zero**; in such case we have the **Ricci flat spacetimes**, $Ric = 0$.
- Spacetimes with Ricci tensor of cosmological type **physicsts** call **vaccuum solutions**; **with** ($\Lambda \neq 0$) or **without** ($\Lambda = 0$) **cosmological constant.**

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + pg_{\mu\nu}$$

with $u^\mu = g^{\mu\nu}u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure**. The timelike unit vector u^μ is the 4-velocity of particles of the fluid.

In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + pg_{\mu\nu}$$

with $u^\mu = g^{\mu\nu}u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure**. The timelike unit vector u^μ is the 4-velocity of particles of the fluid.

In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + pg_{\mu\nu}$$

with $u^\mu = g^{\mu\nu}u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure**. The timelike unit vector u^μ is the 4-velocity of particles of the fluid.

In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + pg_{\mu\nu}$$

with $u^\mu = g^{\mu\nu}u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure.** The timelike unit vector u^μ is the 4-velocity of particles of the fluid.

In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + pg_{\mu\nu}$$

with $u^\mu = g^{\mu\nu}u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure.** The timelike unit vector u^μ is the 4-velocity of particles of the fluid.

In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + p g_{\mu\nu}$$

with $u^\mu = g^{\mu\nu}u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure.** The timelike unit vector u^μ is the 4-velocity of particles of the fluid.
In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + p g_{\mu\nu}$$

with $u^\mu = g^{\mu\nu} u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure.** The timelike unit vector u^μ is the 4-velocity of particles of the fluid.
In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + p g_{\mu\nu}$$

with $u^\mu = g^{\mu\nu} u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure.** The timelike unit vector u^μ is the 4-velocity of particles of the fluid.
In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + pg_{\mu\nu}$$

with $u^\mu = g^{\mu\nu}u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure.** The timelike unit vector u^μ is the 4-velocity of particles of the fluid.

In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + pg_{\mu\nu}$$

with $u^\mu = g^{\mu\nu}u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure.** The timelike unit vector u^μ is the 4-velocity of particles of the fluid.

In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

The energy momentum of an **incompressible fluid** is given by

$$T_{\mu\nu} = (\mu + p)u_\mu u_\nu + p g_{\mu\nu}$$

with $u^\mu = g^{\mu\nu} u_\nu$ a **unit timelike** vector $u^\mu u_\nu = -1$. Here μ is a scalar function describing the **energy density** of the fluid, and p is a scalar function describing its **preasure.** The timelike unit vector u^μ is the 4-velocity of particles of the fluid.
In the frame **comoving with the fluid** we have $u^\mu = (1, 0, 0, 0)$ and $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this frame we have

$$T_{\mu\nu} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \text{ or } T^\mu{}_\nu = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

Thus, the **perfect fluid** energy momentum tensor, is the **next simple after Einstein's**: we have **two real eigenvalues**, one with **multiplicity one** (μ), and one **with multiplicity three** (p).

Energy momentum of perfect fluid

- The system of Einstein's equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$ with energy momentum tensor of a perfect fluid is **underdetermined.**
- Even under very strong symmetry assumptions about g one needs additional equation to solve it.
- The neccessary equation to make the Einstein's system determined is a phenomenological equation called the **equation of state.**
- In its simplest form it gives an implicit relation between μ and p; in GR it is usually given in the form $p = p(\mu)$.

- The system of Einstein's equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$ with energy momentum tensor of a perfect fluid is **underdetermined.**
- Even under very strong symmetry assumptions about g one needs additional equation to solve it.
- The neccessary equation to make the Einstein's system determined is a phenomenological equation called the **equation of state.**
- In its simplest form it gives an implicit relation between μ and p; in GR it is usually given in the form $p = p(\mu)$.

Energy momentum of perfect fluid

- The system of Einstein's equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$ with energy momentum tensor of a perfect fluid is **underdetermined.**
- Even under very strong symmetry assumptions about g one needs additional equation to solve it.
- The neccessary equation to make the Einstein's system determined is a phenomenological equation called the **equation of state.**
- In its simplest form it gives an implicit relation between μ and p; in GR it is usually given in the form $p = p(\mu)$.

- The system of Einstein's equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$ with energy momentum tensor of a perfect fluid is **underdetermined.**
- Even under very strong symmetry assumptions about g one needs additional equation to solve it.
- The neccessary equation to make the Einstein's system determined is a phenomenological equation called the **equation of state.**
- In its simplest form it gives an implicit relation between μ and p; in GR it is usually given in the form $p = p(\mu)$.

- The system of Einstein's equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$ with energy momentum tensor of a perfect fluid is **underdetermined.**
- Even under very strong symmetry assumptions about g one needs additional equation to solve it.
- The neccessary equation to make the Einstein's system determined is a phenomenological equation called the **equation of state.**
- In its simplest form it gives an implicit relation between μ and p; in GR it is usually given in the form $p = p(\mu)$.

- The system of Einstein's equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$ with energy momentum tensor of a perfect fluid is **underdetermined.**
- Even under very strong symmetry assumptions about g one needs additional equation to solve it.
- The neccessary equation to make the Einstein's system determined is a phenomenological equation called the **equation of state.**
- In its simplest form it gives an implicit relation between μ and p; in GR it is usually given in the form $p = p(\mu)$.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

Perfect fluid as a polytrope

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

Perfect fluid as a polytrope

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

The simplest example is when $p = w\mu$.

- if $w = -1$, $p + \mu = 0$, and we recover the cosmological constant case with one eigenvalue μ; $T_{\rho\nu} = pg_{\rho\nu}$;
- if $w = 0$, $p = 0$ - **no preasure**; such fluid is called **dust**; $T_{\rho\nu} = \mu u_\rho u_\nu$.
- if $w = \frac{1}{3}$, $p = \frac{1}{3}\mu$ - and this is a relation known from statistical physics characterizing preasure of **light** carying energy density μ.
- Note that in the **standard cosmology** they believe that the **Universe at the beginning** was **radiation dominated** ($p = \frac{1}{3}\mu$), that **now** it is **matter dominated** ($p = 0$), and **at the end** of its evolution it will be of **cosmological constant type** ($p = -\mu$).
- Other cases discussed in (more fancy) cosmology is $w = -\frac{1}{3}$ - it corresponds to a gas of strings, and $w = -\frac{2}{3}$ - it corresponds to a gas of domain walls.

Electromagnetic field

- The electromagnetic field in vacuum is described in General Relativity by a field of a 2-form $F = \frac{1}{2}F_{\mu\nu}\theta^\mu \wedge \theta^\nu$, satisfying **Maxwell's equations**
$$\mathrm{d}F = \mathrm{d} * F = 0$$
coupled to the Einstein's equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, with Maxwell's energy momentum tensor:
$$T_{\mu\nu} = F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}.$$
- From the algebraic point of view there are **two kinds of energy momentum tensors** of the electromagnetic field. they depend on the fact if the complex 2-form $\tilde{F} = F - i * F$ is **simple**, $\tilde{F} \wedge \tilde{F} = 0$, or **not**.
- If $\tilde{F}$ is **not** simple, the corresponding energy momentum tensor $T^\mu{}_\nu$ has **two real eigenvalues**, each with **multiplicity two**.

- The electromagnetic field in vacuum is described in General Relativity by a field of a 2-form $F = \frac{1}{2} F_{\mu\nu} \theta^\mu \wedge \theta^\nu$, satisfying **Maxwell's equations**
$$\mathrm{d}F = \mathrm{d} * F = 0$$
coupled to the Einstein's equations $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, with Maxwell's energy momentum tensor:
$$T_{\mu\nu} = F_{\mu\rho} F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}.$$
- From the algebraic point of view there are **two kinds of energy momentum tensors** of the electromagnetic field. they depend on the fact if the complex 2-form $\tilde{F} = F - i * F$ is **simple**, $\tilde{F} \wedge \tilde{F} = 0$, or **not**.
- If $\tilde{F}$ is **not** simple, the corresponding energy momentum tensor $T^\mu{}_\nu$ has **two real eigenvalues**, each with **multiplicity two**.

- The electromagnetic field in vacuum is described in General Relativity by a field of a 2-form $F = \frac{1}{2} F_{\mu\nu} \theta^\mu \wedge \theta^\nu$, satisfying **Maxwell's equations**
$$\mathrm{d}F = \mathrm{d} * F = 0$$
coupled to the Einstein's equations $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, with Maxwell's energy momentum tensor:
$$T_{\mu\nu} = F_{\mu\rho} F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}.$$
- From the algebraic point of view there are **two kinds of energy momentum tensors** of the electromagnetic field. they depend on the fact if the complex 2-form $\tilde{F} = F - i * F$ is **simple**, $\tilde{F} \wedge \tilde{F} = 0$, or **not**.
- If $\tilde{F}$ is **not** simple, the corresponding energy momentum tensor $T^\mu{}_\nu$ has **two real eigenvalues**, each with **multiplicity two**.

- The electromagnetic field in vacuum is described in General Relativity by a field of a 2-form $F = \frac{1}{2} F_{\mu\nu} \theta^\mu \wedge \theta^\nu$, satisfying **Maxwell's equations**

$$\mathrm{d}F = \mathrm{d} * F = 0$$

coupled to the Einstein's equations $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, with Maxwell's energy momentum tensor:

$$T_{\mu\nu} = F_{\mu\rho} F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}.$$

- From the algebraic point of view there are **two kinds of energy momentum tensors** of the electromagnetic field. they depend on the fact if the complex 2-form $\tilde{F} = F - i * F$ is **simple**, $\tilde{F} \wedge \tilde{F} = 0$, or **not**.
- If $\tilde{F}$ is **not** simple, the corresponding energy momentum tensor $T^\mu{}_\nu$ has **two real eigenvalues**, each with **multiplicity two**.

- The electromagnetic field in vacuum is described in General Relativity by a field of a 2-form $F = \frac{1}{2}F_{\mu\nu}\theta^\mu \wedge \theta^\nu$, satisfying **Maxwell's equations**
$$\mathrm{d}F = \mathrm{d} * F = 0$$
coupled to the Einstein's equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, with Maxwell's energy momentum tensor:
$$T_{\mu\nu} = F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}.$$
- From the algebraic point of view there are **two kinds of energy momentum tensors** of the electromagnetic field. they depend on the fact if the complex 2-form $\tilde{F} = F - i * F$ is **simple**, $\tilde{F} \wedge \tilde{F} = 0$, or **not**.
- If $\tilde{F}$ is **not** simple, the corresponding energy momentum tensor $T^\mu{}_\nu$ has **two real eigenvalues**, each with **multiplicity two**.

- The electromagnetic field in vacuum is described in General Relativity by a field of a 2-form $F = \frac{1}{2}F_{\mu\nu}\theta^{\mu} \wedge \theta^{\nu}$, satisfying **Maxwell's equations**
$$\mathrm{d}F = \mathrm{d} * F = 0$$
coupled to the Einstein's equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, with Maxwell's energy momentum tensor:
$$T_{\mu\nu} = F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}.$$
- From the algebraic point of view there are **two kinds of energy momentum tensors** of the electromagnetic field. they depend on the fact if the complex 2-form $\tilde{F} = F - i * F$ is **simple**, $\tilde{F} \wedge \tilde{F} = 0$, or **not**.
- If $\tilde{F}$ is **not** simple, the corresponding energy momentum tensor $T^{\mu}{}_{\nu}$ has **two real eigenvalues**, each with **multiplicity two**.

- The electromagnetic field in vacuum is described in General Relativity by a field of a 2-form $F = \frac{1}{2} F_{\mu\nu} \theta^\mu \wedge \theta^\nu$, satisfying **Maxwell's equations**
$$\mathrm{d}F = \mathrm{d} * F = 0$$
coupled to the Einstein's equations $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, with Maxwell's energy momentum tensor:
$$T_{\mu\nu} = F_{\mu\rho} F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}.$$
- From the algebraic point of view there are **two kinds of energy momentum tensors** of the electromagnetic field. they depend on the fact if the complex 2-form $\tilde{F} = F - i * F$ is **simple**, $\tilde{F} \wedge \tilde{F} = 0$, or **not**.
- If $\tilde{F}$ is **not** simple, the corresponding energy momentum tensor $T^\mu{}_\nu$ has **two real eigenvalues**, each with **multiplicity two.**

- The electromagnetic field in vacuum is described in General Relativity by a field of a 2-form $F = \frac{1}{2} F_{\mu\nu} \theta^\mu \wedge \theta^\nu$, satisfying **Maxwell's equations**
$$\mathrm{d}F = \mathrm{d} * F = 0$$
coupled to the Einstein's equations $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$, with Maxwell's energy momentum tensor:
$$T_{\mu\nu} = F_{\mu\rho} F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}.$$
- From the algebraic point of view there are **two kinds of energy momentum tensors** of the electromagnetic field. they depend on the fact if the complex 2-form $\tilde{F} = F - i * F$ is **simple**, $\tilde{F} \wedge \tilde{F} = 0$, or **not**.
- If $\tilde{F}$ is **not** simple, the corresponding energy momentum tensor $T^\mu{}_\nu$ has **two real eigenvalues**, each with **multiplicity two**.

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k) = 0$, and such that $\tilde{F}(k,\cdot) = 0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu} = \Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation**. They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu} = \Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu} = \Phi k_\rho k_\nu$ to $T_{\rho\nu} = \mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k)=0$, and such that $\tilde{F}(k,\cdot)=0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu}=\Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation**. They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu}=\Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu}=\Phi k_\rho k_\nu$ to $T_{\rho\nu}=\mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k)=0$, and such that $\tilde{F}(k,\cdot)=0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu}=\Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation**. They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu}=\Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu}=\Phi k_\rho k_\nu$ to $T_{\rho\nu}=\mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

Null electromagnetic field and pure radiation

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k) = 0$, and such that $\tilde{F}(k,\cdot) = 0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu} = \Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation**. They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu} = \Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu} = \Phi k_\rho k_\nu$ to $T_{\rho\nu} = \mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k) = 0$, and such that $\tilde{F}(k,\cdot) = 0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu} = \Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation**. They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu} = \Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu} = \Phi k_\rho k_\nu$ to $T_{\rho\nu} = \mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k) = 0$, and such that $\tilde{F}(k, \cdot) = 0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu} = \Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation.** They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu} = \Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu} = \Phi k_\rho k_\nu$ to $T_{\rho\nu} = \mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k) = 0$, and such that $\tilde{F}(k,\cdot) = 0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu} = \Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation**. They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu} = \Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu} = \Phi k_\rho k_\nu$ to $T_{\rho\nu} = \mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k)=0$, and such that $\tilde{F}(k,\cdot)=0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu}=\Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation**. They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu}=\Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu}=\Phi k_\rho k_\nu$ to $T_{\rho\nu}=\mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k)=0$, and such that $\tilde{F}(k,\cdot)=0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu}=\Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation**. They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu}=\Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu}=\Phi k_\rho k_\nu$ to $T_{\rho\nu}=\mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

- If $\tilde{F}$ is simple, then there exists a **nonzero real vector field** k on M which is **null**, $g(k,k)=0$, and such that $\tilde{F}(k,\cdot)=0$. In such case, the energy momentum tensor $T_{\mu\nu}$ of the corresponding Maxwell form F is $T_{\mu\nu}=\Phi k_\mu k_\nu$, with Φ nonegative real function on M.
- The electromagnetic field corresponding to a **simple** $\tilde{F}$ is called **null**. Null electromagnetic fields correspond to **pure radiation**. They behave as **plane electromagnetic waves** in Minkowski spacetime.
- One can consider energy momentum tensors of the form $T_{\mu\nu}=\Phi k_\mu k_\nu$ with **null** k_μ **without reference to any electromagnetic field**. Such energy momentum tensors are interpreted as being produced by **incoherent radiation of massless particles propagating with speed of light along the rays of** k_μ. Due to the similarity of $T_{\rho\nu}=\Phi k_\rho k_\nu$ to $T_{\rho\nu}=\mu u_\rho u_\nu$ – the energy momentum of a **dust**, the **pure radiation** is also called **null dust**.

A **physically resonable** energy momentum tensor **has to satisfy** the

dominant energy condition:

the **local energy density** $\rho = T_{\mu\nu}u^\mu u^\nu$ **as measured by any observer** with 4-velocity u_μ is **non-negative** and **the local energy flow** $q^\mu = T^\mu{}_\nu u^\nu$ is **causal**

$$T_{\mu\nu}u^\mu u^\nu \geq 0 \qquad \text{and} \qquad q^\mu q_\mu \leq 0.$$

In particular, for the politrope perfect fluid with $p = w\mu$ this gives $\mu \geq 0$, $|w| \leq 1$.

A **physically resonable** energy momentum tensor **has to satisfy** the

dominant energy condition:

the **local energy density** $\rho = T_{\mu\nu}u^\mu u^\nu$ **as measured by any observer** with 4-velocity u_μ is **non-negative** and **the local energy flow** $q^\mu = T^\mu{}_\nu u^\nu$ is **causal**

$$T_{\mu\nu}u^\mu u^\nu \geq 0 \qquad \text{and} \qquad q^\mu q_\mu \leq 0.$$

In particular, for the politrope perfect fluid with $p = w\mu$ this gives $\mu \geq 0$, $|w| \leq 1$.

A **physically resonable** energy momentum tensor **has to satisfy** the

dominant energy condition:

the **local energy density** $\rho = T_{\mu\nu} u^\mu u^\nu$ **as measured by any observer** with 4-velocity u_μ is **non-negative** and **the local energy flow** $q^\mu = T^\mu{}_\nu u^\nu$ is **causal**

$$T_{\mu\nu} u^\mu u^\nu \geq 0 \qquad \text{and} \qquad q^\mu q_\mu \leq 0.$$

In particular, for the politrope perfect fluid with $p = w\mu$ this gives $\mu \geq 0$, $|w| \leq 1$.

A **physically resonable** energy momentum tensor **has to satisfy** the

dominant energy condition:

the **local energy density** $\rho = T_{\mu\nu}u^\mu u^\nu$ **as measured by any observer** with 4-velocity u_μ is **non-negative** and **the local energy flow** $q^\mu = T^\mu{}_\nu u^\nu$ is **causal**

$$T_{\mu\nu}u^\mu u^\nu \geq 0 \qquad \text{and} \qquad q^\mu q_\mu \leq 0.$$

In particular, for the politrope perfect fluid with $p = w\mu$ this gives $\mu \geq 0$, $|w| \leq 1$.

A **physically resonable** energy momentum tensor **has to satisfy** the

dominant energy condition:

the **local energy density** $\rho = T_{\mu\nu}u^\mu u^\nu$ **as measured by any observer** with 4-velocity u_μ is **non-negative** and **the local energy flow** $q^\mu = T^\mu{}_\nu u^\nu$ is **causal**

$$T_{\mu\nu}u^\mu u^\nu \geq 0 \qquad \text{and} \qquad q^\mu q_\mu \leq 0.$$

In particular, for the politrope perfect fluid with $p = w\mu$ this gives $\mu \geq 0$, $|w| \leq 1$.

A **physically resonable** energy momentum tensor **has to satisfy** the

dominant energy condition:

the **local energy density** $\rho = T_{\mu\nu}u^\mu u^\nu$ **as measured by any observer** with 4-velocity u_μ is **non-negative** and **the local energy flow** $q^\mu = T^\mu{}_\nu u^\nu$ is **causal**

$$T_{\mu\nu}u^\mu u^\nu \geq 0 \qquad \text{and} \qquad q^\mu q_\mu \leq 0.$$

In particular, for the politrope perfect fluid with $p = w\mu$ this gives $\mu \geq 0$, $|w| \leq 1$.

- A spacetime M with a conformal class of metrics $[g]$ i.e. with Lorentzian metrics such that any two g_1 and g_2 are related to each other via $g_2 = e^{2\phi} g_1$ is called a **conformal spacetime** $(M, [g])$.
- **Problem**:
  Characterize conformal spacetimes having in conformal class metrics whose energy momentum tensors, via Einstein equations, are in a given algebraic type.
- The **answer is known** for **Einstein metrics** and **pure radiation.**
- I would like to have an answer for **perfect fluids (???)**, and in particular for **various politropes (???).**

- A spacetime M with a conformal class of metrics $[g]$ i.e. with Lorentzian metrics such that any two g_1 and g_2 are related to each other via $g_2 = \mathrm{e}^{2\phi} g_1$ is called a **conformal spacetime** $(M, [g])$.
- **Problem:**
  Characterize conformal spacetimes having in conformal class metrics whose energy momentum tensors, via Einstein equations, are in a given algebraic type.
- The **answer is known** for **Einstein metrics** and **pure radiation.**
- I would like to have an answer for **perfect fluids (???)**, and in particular for **various politropes (???).**

- A spacetime M with a conformal class of metrics $[g]$ i.e. with Lorentzian metrics such that any two g_1 and g_2 are related to each other via $g_2 = \mathrm{e}^{2\phi} g_1$ is called a **conformal spacetime** $(M, [g])$.
- **Problem**:
  Characterize conformal spacetimes having in conformal class metrics whose energy momentum tensors, via Einstein equations, are in a given algebraic type.
- The **answer is known** for **Einstein metrics** and **pure radiation.**
- I would like to have an answer for **perfect fluids (???)**, and in particular for **various politropes (???).**

Conformal spacetimes

- A spacetime M with a conformal class of metrics $[g]$ i.e. with Lorentzian metrics such that any two g_1 and g_2 are related to each other via $g_2 = \mathrm{e}^{2\phi} g_1$ is called a **conformal spacetime** $(M, [g])$.
- **Problem**:
  Characterize conformal spacetimes having in conformal class metrics whose energy momentum tensors, via Einstein equations, are in a given algebraic type.
- The **answer is known** for **Einstein metrics** and **pure radiation.**
- I would like to have an answer for **perfect fluids (???)**, and in particular for **various politropes (???).**

- A spacetime M with a conformal class of metrics $[g]$ i.e. with Lorentzian metrics such that any two g_1 and g_2 are related to each other via $g_2 = \mathrm{e}^{2\phi} g_1$ is called a **conformal spacetime** $(M, [g])$.
- **Problem**:
  Characterize conformal spacetimes having in conformal class metrics whose energy momentum tensors, via Einstein equations, are in a given algebraic type.
- The **answer is known** for **Einstein metrics** and **pure radiation.**
- I would like to have an answer for **perfect fluids (???)**, and in particular for **various politropes (???).**

- A spacetime M with a conformal class of metrics $[g]$ i.e. with Lorentzian metrics such that any two g_1 and g_2 are related to each other via $g_2 = \mathrm{e}^{2\phi} g_1$ is called a **conformal spacetime** $(M, [g])$.
- **Problem**:
  Characterize conformal spacetimes having in conformal class metrics whose energy momentum tensors, via Einstein equations, are in a given algebraic type.
- The **answer is known** for **Einstein metrics** and **pure radiation.**
- I would like to have an answer for **perfect fluids (???)**, and in particular for **various politropes (???)**.

- A spacetime M with a conformal class of metrics $[g]$ i.e. with Lorentzian metrics such that any two g_1 and g_2 are related to each other via $g_2 = \mathrm{e}^{2\phi} g_1$ is called a **conformal spacetime** $(M, [g])$.
- **Problem**:
  Characterize conformal spacetimes having in conformal class metrics whose energy momentum tensors, via Einstein equations, are in a given algebraic type.
- The **answer is known** for **Einstein metrics** and **pure radiation.**
- I would like to have an answer for **perfect fluids (???)**, and in particular for **various politropes (???).**

- A spacetime M with a conformal class of metrics $[g]$ i.e. with Lorentzian metrics such that any two g_1 and g_2 are related to each other via $g_2 = \mathrm{e}^{2\phi} g_1$ is called a **conformal spacetime** $(M, [g])$.
- **Problem**:
  Characterize conformal spacetimes having in conformal class metrics whose energy momentum tensors, via Einstein equations, are in a given algebraic type.
- The **answer is known** for **Einstein metrics** and **pure radiation.**
- I would like to have an answer for **perfect fluids (???)**, and in particular for **various politropes (???).**

- Classification of energy momentum tensors in spacetime:
  - Plebański J (1964) *The algebraic structure of the tensor of matter*, Acta Phys. Polon. **26**, 963-...
- Conformal characterizations of spacetimes with a given $T_{\mu\nu}$:
  - Kozameh C N, Newman E T and Tod K P (1985) *Conformal Einstein spaces* Gen. Rel. Grav. **17** 343–352
  - Kozameh C N, Newman E T and Nurowski P (2003) *Conformal Einstein equations and Cartan conformal connection* Class. Q. Grav. **20** 3029 – 3035
  - Gover A R and Nurowski P (2006) *Obstructions to conformally Einstein metrics in n dimensions* Journ. Geom. Phys. **56** 450 – 484
  - Leistner Th and Nurowski P (2012) *Conformal pure radiation with parallel rays*, Class. Q. Grav. **29**, 055007 (15pp), doi:10.1088/0264-9381/29/5/055007

- Classification of energy momentum tensors in spacetime:
  - Plebański J (1964) *The algebraic structure of the tensor of matter*, Acta Phys. Polon. **26**, 963-...
- Conformal characterizations of spacetimes with a given $T_{\mu\nu}$:
  - Kozameh C N, Newman E T and Tod K P (1985) *Conformal Einstein spaces* Gen. Rel. Grav. **17** 343–352
  - Kozameh C N, Newman E T and Nurowski P (2003) *Conformal Einstein equations and Cartan conformal connection* Class. Q. Grav. **20** 3029 – 3035
  - Gover A R and Nurowski P (2006) *Obstructions to conformally Einstein metrics in n dimensions* Journ. Geom. Phys. **56** 450 – 484
  - Leistner Th and Nurowski P (2012) *Conformal pure radiation with parallel rays*, Class. Q. Grav. **29**, 055007 (15pp), doi:10.1088/0264-9381/29/5/055007

- Classification of energy momentum tensors in spacetime:
  - Plebański J (1964) *The algebraic structure of the tensor of matter*, Acta Phys. Polon. **26**, 963-...
- Conformal characterizations of spacetimes with a given $T_{\mu\nu}$:
  - Kozameh C N, Newman E T and Tod K P (1985) *Conformal Einstein spaces* Gen. Rel. Grav. **17** 343–352
  - Kozameh C N, Newman E T and Nurowski P (2003) *Conformal Einstein equations and Cartan conformal connection* Class. Q. Grav. **20** 3029 – 3035
  - Gover A R and Nurowski P (2006) *Obstructions to conformally Einstein metrics in n dimensions* Journ. Geom. Phys. **56** 450 – 484
  - Leistner Th and Nurowski P (2012) *Conformal pure radiation with parallel rays*, Class. Q. Grav. **29**, 055007 (15pp), doi:10.1088/0264-9381/29/5/055007

- Classification of energy momentum tensors in spacetime:
  - Plebański J (1964) *The algebraic structure of the tensor of matter*, Acta Phys. Polon. **26**, 963-...
- Conformal characterizations of spacetimes with a given $T_{\mu\nu}$:
  - Kozameh C N, Newman E T and Tod K P (1985) *Conformal Einstein spaces* Gen. Rel. Grav. **17** 343–352
  - Kozameh C N, Newman E T and Nurowski P (2003) *Conformal Einstein equations and Cartan conformal connection* Class. Q. Grav. **20** 3029 – 3035
  - Gover A R and Nurowski P (2006) *Obstructions to conformally Einstein metrics in n dimensions* Journ. Geom. Phys. **56** 450 – 484
  - Leistner Th and Nurowski P (2012) *Conformal pure radiation with parallel rays*, Class. Q. Grav. **29**, 055007 (15pp), doi:10.1088/0264-9381/29/5/055007

- Classification of energy momentum tensors in spacetime:
  - Plebański J (1964) *The algebraic structure of the tensor of matter*, Acta Phys. Polon. **26**, 963-...
- Conformal characterizations of spacetimes with a given $T_{\mu\nu}$:
  - Kozameh C N, Newman E T and Tod K P (1985) *Conformal Einstein spaces* Gen. Rel. Grav. **17** 343–352
  - Kozameh C N, Newman E T and Nurowski P (2003) *Conformal Einstein equations and Cartan conformal connection* Class. Q. Grav. **20** 3029 – 3035
  - Gover A R and Nurowski P (2006) *Obstructions to conformally Einstein metrics in n dimensions* Journ. Geom. Phys. **56** 450 – 484
  - Leistner Th and Nurowski P (2012) *Conformal pure radiation with parallel rays*, Class. Q. Grav. **29**, 055007 (15pp), doi:10.1088/0264-9381/29/5/055007

Literature

- Classification of energy momentum tensors in spacetime:
  - Plebański J (1964) *The algebraic structure of the tensor of matter*, Acta Phys. Polon. **26**, 963-...
- Conformal characterizations of spacetimes with a given $T_{\mu\nu}$:
  - Kozameh C N, Newman E T and Tod K P (1985) *Conformal Einstein spaces* Gen. Rel. Grav. **17** 343–352
  - Kozameh C N, Newman E T and Nurowski P (2003) *Conformal Einstein equations and Cartan conformal connection* Class. Q. Grav. **20** 3029 – 3035
  - Gover A R and Nurowski P (2006) *Obstructions to conformally Einstein metrics in n dimensions* Journ. Geom. Phys. **56** 450 – 484
  - Leistner Th and Nurowski P (2012) *Conformal pure radiation with parallel rays*, Class. Q. Grav. **29**, 055007 (15pp), doi:10.1088/0264-9381/29/5/055007

- Classification of energy momentum tensors in spacetime:
  - Plebański J (1964) *The algebraic structure of the tensor of matter*, Acta Phys. Polon. **26**, 963-...
- Conformal characterizations of spacetimes with a given $T_{\mu\nu}$:
  - Kozameh C N, Newman E T and Tod K P (1985) *Conformal Einstein spaces* Gen. Rel. Grav. **17** 343–352
  - Kozameh C N, Newman E T and Nurowski P (2003) *Conformal Einstein equations and Cartan conformal connection* Class. Q. Grav. **20** 3029 – 3035
  - Gover A R and Nurowski P (2006) *Obstructions to conformally Einstein metrics in n dimensions* Journ. Geom. Phys. **56** 450 – 484
  - Leistner Th and Nurowski P (2012) *Conformal pure radiation with parallel rays*, Class. Q. Grav. **29**, 055007 (15pp), doi:10.1088/0264-9381/29/5/055007

- Classification of energy momentum tensors in spacetime:

  - Plebański J (1964) *The algebraic structure of the tensor of matter*, Acta Phys. Polon. **26**, 963-...

- Conformal characterizations of spacetimes with a given $T_{\mu\nu}$:

  - Kozameh C N, Newman E T and Tod K P (1985) *Conformal Einstein spaces* Gen. Rel. Grav. **17** 343–352
  - Kozameh C N, Newman E T and Nurowski P (2003) *Conformal Einstein equations and Cartan conformal connection* Class. Q. Grav. **20** 3029 – 3035
  - Gover A R and Nurowski P (2006) *Obstructions to conformally Einstein metrics in n dimensions* Journ. Geom. Phys. **56** 450 – 484
  - Leistner Th and Nurowski P (2012) *Conformal pure radiation with parallel rays*, Class. Q. Grav. **29**, 055007 (15pp), doi:10.1088/0264-9381/29/5/055007