Applications of Tractor Calculus in General Relativity

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Asymptotically de Sitter spacetimes

Einstein field equations

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = T_{ab}, \tag{1}$$

with $\Lambda > 0$ and

$$g_{ab} = \sigma^{-2} \mathbf{g}_{ab}, \quad T_{ab} = \sigma^{q} \tau_{ab}, \quad q \geq 0. \tag{2}$$

Characterized the geometry of conformal infinity Σ ($n = 4$) in terms of constraints relating conformal fundamental forms

$$\mathring{K}_{ab}, \ W_{\hat{n}a\hat{n}b}, \ \overline{\nabla}_b A^{\top}_{a\hat{n}}{}^{b}, \ \mathring{\top} B_{ab}$$

and the stress-energy tensor density τ_{ab}.

Example:

Friedmann–Lemaître–Robertson–Walker (FLRW) metric

We want to solve $(n = 4, \Lambda = 0)$

$$\widetilde{R}_{ab} - \frac{1}{2}\widetilde{R}\widetilde{g}_{ab} = \widetilde{T}_{ab}, \tag{3}$$

with

$$\widetilde{g} = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right) \tag{4}$$

and the *perfect fluid* stress-energy tensor:

$$\widetilde{T}_{ab} = (\rho + p)\, v_a v_b + p\widetilde{g}_{ab}, \tag{5}$$

where $\rho = \rho(t)$ is the density, $p = p(t)$ is the pressure and $v \equiv \partial_t$ is the four-velocity.

Equation of state:

$$p = w\rho, \quad -\frac{1}{3} \leq w \leq 1 \tag{6}$$

Spacetimes with initial isotropic singularity

Solution with initial singularity at $t = 0 \implies a(0) = 0$:

$$a = c_1 t^{\frac{2}{3(w+1)}}, \quad \rho = c_2 t^{-2}, \quad c_1, c_2 = \mathrm{const.} \tag{7}$$

so

$$\widetilde{g} = -dt^2 + c_1^2 t^{\frac{4}{3(w+1)}} \left(dx^2 + dy^2 + dz^2\right). \tag{8}$$

Introduce new time coordinate τ defined by

$$\frac{dt}{c_1 t^{\frac{2}{3(w+1)}}} = d\tau \tag{9}$$

Then

$$\begin{aligned} \widetilde{g} &= c_3 \tau^{\frac{4}{3w+1}} \left(-d\tau^2 + dx^2 + dy^2 + dz^2\right) \\ &= \Omega^{\alpha} \left(-d\tau^2 + dx^2 + dy^2 + dz^2\right), \quad \alpha = \frac{4}{3w+1} > 0, \end{aligned} \tag{10}$$

if $\left(w \neq -\frac{1}{3}\right)$ and the initial isotropic singularity can be defined as a spacelike hypersurface where $\Omega = 0$.

Conformal geometry of initial singularity

A. R. Gover, J. Kopiński and A. Waldron, *The geometry of an isotropic Big Bang*, coming soon.

Isotropic singularity spacetime (ISS): n-dimensional spacetime $\left(\widetilde{M}, \widetilde{g}_{ab}\right)$ that arises as follows. There is a smooth manifold M with equipped with

- smooth conformal structure $\mathbf{c}$ of Lorentzian signature
- spacelike boundary Σ
- a scale $\tau \in \Gamma\left(\mathcal{E}[u]\right)$ with $u < 0$ that is defining:
  - $\Sigma = \tau^{-1}(0)$
  - $\nabla^g_a \tau \neq 0$ on Σ

Then $\widetilde{M} := \{x \in M \big| \tau(x) > 0\}$ and

$$\widetilde{g}_{ab} := \tau^{\alpha}\mathbf{g}_{ab}, \quad \alpha := -\frac{2}{u}, \tag{11}$$

i.e. the physical metric $\widetilde{g}_{ab}$ is degenerate on Σ.

Einstein field equations

Let $\left(\widetilde{M}, \widetilde{g}_{ab}\right)$ satisfy the Einstein field equations with cosmological constant Λ,

$$\widetilde{R}_{ab} - \frac{1}{2}\widetilde{R}\widetilde{g}_{ab} + \Lambda\widetilde{g}_{ab} = \widetilde{T}_{ab}, \tag{12}$$

where $\widetilde{T}_{ab}$ is the stress-energy tensor. After splitting into trace-free and trace parts,

$$\mathring{\widetilde{P}}_{ab} = \frac{1}{n-2}\mathring{\widetilde{T}}_{ab}, \quad \widetilde{R} = \frac{2}{n-2}\left(n\Lambda - \widetilde{T}_c{}^c\right) \tag{13}$$

Let χ_{ab} be the stress-energy tensor density of weight v,

$$\chi_{ab} \in \Gamma\left(\mathcal{E}_{(ab)}[v]\right), \tag{14}$$

i.e.

$$\chi_{ab} = \widetilde{\sigma}^v \widetilde{T}_{ab}, \tag{15}$$

where $\widetilde{g}_{ab} = \widetilde{\sigma}^{-2}\mathbf{g}_{ab}$.

Trace-free part of the Einstein field equations

$$\nabla_a \nabla_b \widetilde{\sigma} + \widetilde{\sigma} P_{ab} - \frac{1}{n}\mathbf{g}_{ab}\left(\Delta\widetilde{\sigma} + \widetilde{\sigma} J\right) = \frac{\widetilde{\sigma}^{1-v}}{n-2}\mathring{\chi}_{ab}. \qquad (16)$$

Let $\tau \in \Gamma\left(\mathcal{E}[u]\right)$ be a defining density of the boundary Σ and

$$\tau := \widetilde{\sigma}^u \implies \widetilde{g}_{ab} = \tau^{-\frac{2}{u}}\mathbf{g}_{ab}, \quad u < 0. \qquad (17)$$

Then equation (16) can be written as

$$-\frac{u-1}{u^2}\nabla_{(a}\tau\nabla_{b)_\circ}\tau + \frac{\tau}{u}\nabla_{(a}\nabla_{b)_\circ}\tau + \tau^2\mathring{P}_{ab} = \tau^{2-\frac{v}{u}}\frac{\mathring{\chi}_{ab}}{n-2}. \qquad (18)$$

Regularity on Σ and $\nabla_a\tau \neq 0\big|_\Sigma$ implies

$$2 - \frac{v}{u} = 0 \implies v = 2u \implies \chi_{ab} \in \Gamma\left(\mathcal{E}_{(ab)}[2u]\right). \qquad (19)$$

Ultimately

$$-\frac{u-1}{u^2}\nabla_{(a}\tau\nabla_{b)_\circ}\tau + \frac{\tau}{u}\nabla_{(a}\nabla_{b)_\circ}\tau + \tau^2\mathring{P}_{ab} = \frac{\mathring{\chi}_{ab}}{n-2}. \qquad (20)$$

Trace part of the Einstein field equations

$$\widetilde{R} = \frac{2}{n-2}\left(n\Lambda - \widetilde{T}_c{}^c\right) \tag{21}$$

equivalent to

$$I_\tau^2 = \frac{2u^2}{(n+2u-2)(n-1)(n-2)}\left(\chi - n\tau^{2-\frac{2}{u}}\Lambda\right), \tag{22}$$

If

$$n + 2u - 2 = 0 \iff u = 1 - \frac{n}{2} \tag{23}$$

then (21) implies

$$\tau\left[\Delta\tau + \left(\frac{2-n}{2}\right)\tau J\right] = \frac{1}{2(n-1)}\left(\chi - n\tau^{\frac{2n}{n-2}}\Lambda\right) \tag{24}$$

and $\chi \overset{\Sigma}{=} 0$.

Einstein field equations and ISS

Genericaly $\left(u \neq 1 - \frac{n}{2}\right)$

$$I_\tau^2 = \frac{2u^2}{(n+2u-2)(n-1)(n-2)}\left(\chi - n\tau^{2-\frac{2}{u}}\Lambda\right), \tag{25}$$

implies

$$\mathbf{g}(n,n) = c_1\chi + \mathcal{O}(\tau) \tag{26}$$

where

$$n_a := \nabla_a \tau \tag{27}$$

is the extension of the normal vector of Σ to M.

Consequences:

- unlike in the case of asymptotically de Sitter spacetimes, the sign of cosmological constant Λ does **not** control the causal character of Σ
- vanishing stress-energy tensor $\implies$ Σ is a null hypersurface

Summary

The Einstein field equations are equivalent to

$$\nabla_a^{\mathcal{T}}\left(I_{\tau^{\frac{1}{u}}}\right) = \frac{\tau^{\frac{1}{u}-2}}{n-2}\mathring{\chi}, \tag{28}$$

and

$$I_\tau^2 = \frac{2u^2}{(n+2u-2)(n-1)(n-2)}\left(\chi - n\tau^{2-\frac{2}{u}}\Lambda\right). \tag{29}$$

The canonical metric of isotropic singularity

Theorem

In any isotropic singularity spacetime the initial hypersurface Σ has a canonical Riemannian metric g_Σ.

Proof.

$\mathbf{g}^{ab}\nabla_a\tau\nabla_b\tau < 0$ has weight $2(u-1) \neq 0$. Hence

$$g_\tau := \left(\mathbf{g}^{ab}\nabla_a\tau\nabla_b\tau\right)^{\frac{1}{1-u}}\mathbf{g} \tag{30}$$

and $g_\Sigma \overset{\Sigma}{:=} g_\tau|_{T\Sigma}$. □

Corollary

Given an isotropic singularity spacetime $(M, \mathbf{g}_{ab}, \tau)$ with $\Sigma = \tau^{-1}(0)$ closed, there is canonically a volume of initial singularity,

$$V_\Sigma := \int_\Sigma dV_{g_\Sigma}. \tag{31}$$

Conformal fundamental forms of isotropic singularity Σ

Extension of the trace-free extrinsic curvature

Let

$$E^{\tau} := \tau^{2-\frac{1}{u}} q^{*} \left(\nabla^{\mathcal{T}} I_{\tau^{\frac{1}{u}}} \right), \tag{32}$$

where q^{*} extracts the middle slot from a tractor, i.e.

$$E^{\tau}_{ab} := -\frac{u-1}{u^2} \nabla_{(a} \tau \nabla_{b)_\circ} \tau + \frac{\tau}{u} \nabla_{(a} \nabla_{b)_\circ} \tau + \tau^2 \mathring{P}_{ab}. \tag{33}$$

The trace-free part of the Einstein field equations implies

$$E^{\tau}_{ab} = \frac{\mathring{\chi}_{ab}}{n-2}. \tag{34}$$

E^{τ}_{ab} and the extension of the unit normal vector $\hat{n}_a$

Let σ be a singular Yamabe scale corresponding to the isotropic singularity Σ, i.e.

$$\Sigma = \sigma^{-1}(0), \quad \nabla_a \sigma|_{\Sigma} \neq 0 \tag{35}$$

and

$$I^2_{\sigma} = -1 + \mathcal{O}(\sigma^n) \tag{36}$$

Conformal fundamental forms of isotropic singularity Σ

Then

$$\mathbf{g}\left(\hat{n},\hat{n}\right) = -1 + \mathcal{O}\left(\sigma^{n}\right) \tag{37}$$

where

$$\widehat{n}_a := \nabla_a \sigma \tag{38}$$

is the extension of the unit normal vector.

Let $\kappa \in \Gamma\left(\mathcal{E}[1-u]\right)$ such that

$$\tau = \frac{\sigma}{\kappa}. \tag{39}$$

Extracting conformal fundamental forms from E^{τ}_{ab}:

a) replace τ by $\sigma\kappa^{-1}$ in E^{τ}_{ab} to get

$$E^{\tau}_{ab} = -\frac{u-1}{u^2\kappa^2}\widehat{n}_{(a}\widehat{n}_{b)_0} + \frac{\sigma}{u\kappa^2}\nabla_{(a}\widehat{n}_{b)_0} + \mathcal{O}\left(\sigma\right) \tag{40}$$

Conformal fundamental forms of isotropic singularity Σ

b) apply $\delta = \nabla_{\hat{n}} + \dots$ to have $\nabla_{(a}\hat{n}_{b)_0}$ in the leading term in σ:

$$\delta E^{\tau}_{ab} = -\frac{1}{u\kappa^2}\nabla_{(a}\hat{n}_{b)_0} - \delta\left(\frac{u-1}{u^2\kappa^2}\hat{n}_{(a}\hat{n}_{b)_0}\right) + \mathcal{O}(\sigma) \qquad (41)$$

b) apply standard definition of conformal fundamental forms with respect to δE^{τ}_{ab}:

$$\mathring{K}^{(i+2)}_{ab} := \mathring{\top}\delta^i\left(\delta E^{\tau}_{ab}\right) \qquad (42)$$

Constraints relating conformal fundamental forms and the stress-energy tensor density on isotropic singularity Σ

We have

$$E^{\tau}_{ab} = \frac{\mathring{\chi}_{ab}}{n-2} \qquad (43)$$

so

$$\mathring{K}^{(i+2)}_{ab} \stackrel{\Sigma}{=} \frac{1}{n-2}\mathring{\top}\delta^i\left(\delta\mathring{\chi}_{ab}\right). \qquad (44)$$

Stress-energy tensor from geometry

Isotropic singularity spacetime

The metric of isotropic singularity spacetime $\widetilde{g}_{ab}$ has the following form,

$$\widetilde{g}_{ab} = \tau^{-\frac{2}{u}} \mathbf{g}_{ab}, \quad \tau \in \Gamma\left(\mathcal{E}[u]\right) \tag{45}$$

and initial singularity is a hypersurface $\Sigma = \tau^{-1}(0)$.

Singular Yamabe scale and κ

There is a canonical singular Yamabe scale σ corresponding to Σ, i.e.

$$\Sigma = \sigma^{-1}(0), \quad I_\sigma^2 = -1 + \mathcal{O}\left(\sigma^n\right). \tag{46}$$

Hence, there exist $\kappa \in \Gamma\left(\mathcal{E}[1-u]\right)$ such that

$$\tau = \frac{\sigma}{\kappa} \tag{47}$$

and $\kappa \neq 0$ everywhere. Ultimately,

$$\left(\kappa, \mathbf{g}_{ab}\right) \longrightarrow \widetilde{g}_{ab} \xrightarrow{\text{EFEs}} \widetilde{T}_{ab}\ \left(\chi_{ab}\right). \tag{48}$$

Admissible stress-energy tensors: energy conditions

Generalizations of the statement 'the energy density of a region of spacetime cannot be negative'

- null energy condition

$$\widetilde{T}_{ab}k^a k^b \geq 0 \quad \text{for every null } k^a$$

- weak energy condition

$$\widetilde{T}_{ab}v^a v^b \geq 0 \quad \text{for every timelike } v^a$$

- dominant energy condition

$$-\widetilde{T}^a{}_b Y^b \text{ is timelike or null for every timelike or null } Y^a$$

- strong energy condition

$$\left(\widetilde{T}_{ab} - \frac{1}{n-2}\widetilde{T}\widetilde{g}_{ab}\right) v^a v^b \geq 0 \quad \text{for every timelike } v^a \tag{49}$$

Conformal Cyclic Cosmology

Introduction

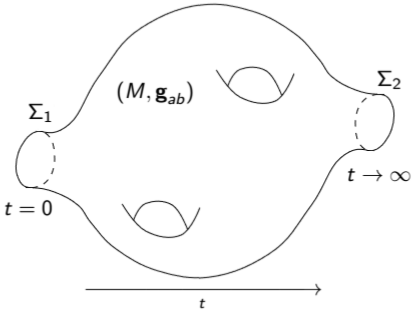
Observations of the Universe suggest that:

1. Universe started with a Big Bang
2. the cosmological constant Λ is positive

Implications:

1. initial state of the Universe can be modelled by the isotropic singularity spacetime
2. the end state of the evolution of the Universe can be modelled by the asymptotically de Sitter spacetime

Conformal extension $(M, \mathbf{g}_{ab})$ of the Universe

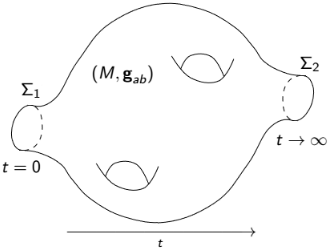


Spacetime (physical) metric $\widehat{g}_{ab}$ satisfies the Einstein field equations

$$\widehat{R}_{ab} - \frac{1}{2}\widehat{R}\widehat{g}_{ab} + \widehat{\Lambda}\widehat{g}_{ab} = \widehat{T}_{ab}, \tag{50}$$

in the interior of M.

Conformal extension $(M, \mathbf{g}_{ab})$ of the Universe



Moreover:

- $\tau \in \Gamma(\mathcal{E}[u])$ is a defining density of Σ_1 and

$$\hat{g}_{ab} = \tau^{-\frac{2}{u}} \mathbf{g}_{ab}, \quad \hat{T}_{ab} = \tau^{-2} \chi_{ab}, \quad u < 0 \tag{51}$$

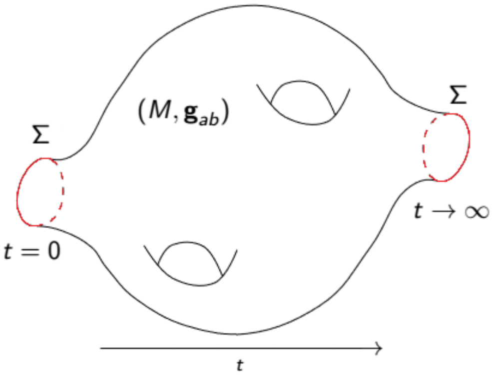
  in a tubular neighbourhood of Σ_1.
- $\sigma \in \Gamma(\mathcal{E}[1])$ is a defining density of Σ_2 and

$$\hat{g}_{ab} = \sigma^{-2} \mathbf{g}_{ab}, \quad \hat{T}_{ab} = \sigma^{q} \tau_{ab}, \quad q \geq 0 \tag{52}$$

  in a tubular neighbourhood of Σ_2.

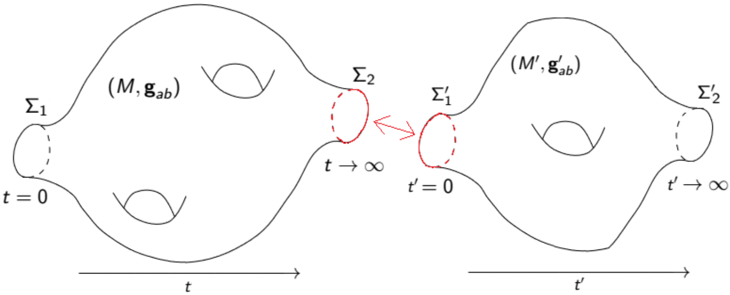
Conformal Periodic Cosmology model

Glue Σ_1 and Σ_2 together (both spacelike hypersurfaces) identifying them as a single hypersurface Σ



Conformal Cyclic Cosmology model

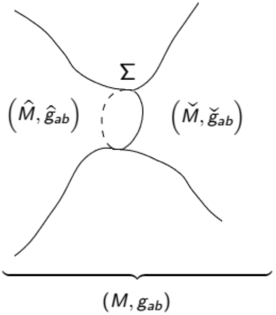
Identify Σ_2 with the isotropic singularity Σ_1' of the conformal extension $(M', \mathbf{g}'_{ab})$ corresponding to the other spacetime $\left(\check{M}, \check{g}_{ab}\right)$



i.e. the end state of the evolution of our Universe (current aeon) is identified with the initial state of the next Universe (next aeon).

P. Tod, *The equations of Conformal Cyclic Cosmology*, Gen. Relativ. Gravit. 47, 17 (2015).

Geometric picture



We have three manifolds with metrics:

- current aeon $\left(\widehat{M}, \widehat{g}_{ab}\right)$
- next aeon $\left(\check{M}, \check{g}_{ab}\right)$
- conformal extension of both (M, g_{ab}) such that

$$\widehat{g}_{ab} = \widehat{\Omega}^2 g_{ab}, \quad \check{g}_{ab} = \check{\Omega}^2 g_{ab} \qquad (53)$$

The metric g_{ab} is called the bridging metric, $M = \widehat{M} \cup \check{M} \cup \Sigma$ and

$$\Sigma = \{\check{\Omega} = 0\} = \{\widehat{\Omega}^{-1} = 0\} \qquad (54)$$

Reciprocal hypothesis

We have

$$\check{g}_{ab} = \check{\Omega}^2 g_{ab} = \check{\Omega}^2 \left(\frac{\hat{g}_{ab}}{\hat{\Omega}^2} \right) = \left(\frac{\check{\Omega}}{\hat{\Omega}} \right)^2 \hat{g}_{ab} \tag{55}$$

Let

$$\check{\Omega}\hat{\Omega} = -1. \tag{56}$$

Then

$$\check{g}_{ab} = \hat{\Omega}^{-4} \hat{g}_{ab} \tag{57}$$

i.e. the metric in the next aeon is determined by the metric in the current aeon given a unique $\hat{\Omega}$.

Equivalently – assume that we know $\hat{g}_{ab}$ and $\hat{\Omega}$. Then

$$g_{ab} = \hat{\Omega}^{-2} \hat{g}_{ab} \tag{58}$$

is known. If $\check{\Omega} = \hat{\Omega}^{-1}$ then

$$\check{g}_{ab} = \hat{\Omega}^{-2} g_{ab}. \tag{59}$$

Friedmann–Lemaître–Robertson–Walker metric

Let

$$\widehat{g} = \hat{a}^2(\tau)\left(-d\tau^2 + dx^2 + dy^2 + dz^2\right) \tag{60}$$

with the perfect fluid stress-energy tensor with four-velocity $\widehat{v} = \partial_t$ and radiation equation of state, $\widehat{p} = \frac{1}{3}\widehat{\rho}$.

Einstein field equations reduce to $\widehat{\rho} = \widehat{m}\widehat{a}^{-4}$, $\widehat{m}$ =const and

$$\left(\frac{d\hat{a}}{d\tau}\right)^2 = \frac{\widehat{m}}{3} + \frac{\hat{\Lambda}}{3}\hat{a}^4 \tag{61}$$

We have

$$\widehat{g}_{ab} = \widehat{\Omega}^2 g_{ab}, \tag{62}$$

so the obvious choice for $\widehat{\Omega}$ is

$$\widehat{\Omega} = c_1\hat{a}. \tag{63}$$

Conformal Cyclic Cosmology – example ($n = 4$)

We have

$$\begin{aligned} \check{g} &= \widehat{\Omega}^{-4}\widehat{g} \\ &= \check{a}^2\left(-d\tau^2 + dx^2 + dy^2 + dz^2\right) \end{aligned} \tag{64}$$

where

$$\check{a} := -\frac{1}{c_1^2\widehat{a}} \tag{65}$$

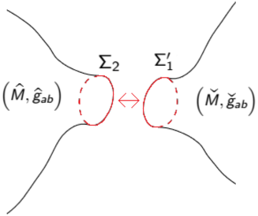
If

$$c_1 = \left(\hat{\Lambda}/\hat{m}\right)^{1/4} \tag{66}$$

then

$$\left(\frac{d\check{a}}{d\tau}\right)^2 = \frac{\check{m}}{3} + \frac{\check{\Lambda}}{3}\check{a}^4 \tag{67}$$

with $\check{m} = \hat{m}$ and $\check{\Lambda} = \hat{\Lambda}$, i.e. $\check{g}$ satisfies the same equation as $\hat{g}$ (aeons are diffeomorphic).



- asymptotically de Sitter spacetime $\left(\hat{M}, \hat{g}_{ab}\right)$,

$$\hat{g}_{ab} = \sigma^{-2}\mathbf{g}_{ab}, \ \hat{T}_{ab} = \sigma^{q}\tau_{ab}, \quad q \geq 0, \tag{68}$$

  and σ is a defining density of Σ_2
- spacetime with isotropic singularity $\left(\check{M}, \check{g}_{ab}\right)$,

$$\check{g}_{ab} = \tau^{-\frac{2}{u}}\mathbf{g}'_{ab}, \ \check{T}_{ab} = \tau^{-2}\chi_{ab}, \quad u < 0, \tag{69}$$

  and τ is a defining density of Σ'_1.

When can Σ_2 and Σ'_1 be identified?
What are the matching conditions?

Simplest model of a spherically symmetric star:
A spherical cluster of matter in an empty spacetime

- interior: homogeneous and isotropic distribution of matter $\rightarrow$ perfect fluid spacetime
- exterior: vacuum $\rightarrow$ Schwarzschild spacetime

We have

$$g_{int} = -dt^2 + a^2(t)\left(dr^2 + \sin^2 r \; g_{S^2}\right) \tag{70}$$

and

$$g_{ext} = -\left(1 - \frac{2m}{r'}\right) dt'^2 + \frac{d(r')^2}{1 - \frac{2m}{r'}} + \left(r'\right)^2 g_{S^2} \tag{71}$$

Let $t = t'$ and consider $t =$const hyperurface. Induced metrics are

$$\overline{g}_{int} = a^2\left(dr^2 + \sin^2 r \; g_{S^2}\right), \qquad \overline{g}_{ext} = \frac{d(r')^2}{1 - \frac{2m}{r'}} + \left(r'\right)^2 g_{S^2}. \tag{72}$$

Let the boundary of a spherical cluster be located at

$$r = R_1, \quad r' = R_2 > 2m \tag{73}$$

(outside of the event horizon of a black hole).

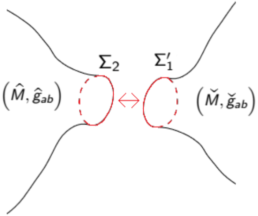
Matching conditions

$$\overline{g}_{int}\Big|_{r=R_1} = \overline{g}_{ext}\Big|_{r'=R_2}, \quad K_{int}\Big|_{r=R_1} = K_{out}\Big|_{r'=R_2} \tag{74}$$

so

$$R_2 = a \sin R_1, \quad \sin R_1 = \sqrt{2\frac{m}{R_2}} \tag{75}$$

which implies $m = \frac{a \sin^3 R_1}{2}$.



- asymptotically de Sitter spacetime $\left(\hat{M}, \hat{g}_{ab}\right)$,

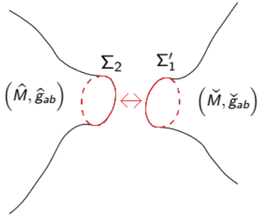
$$\mathring{K}_{ab}^{(i+2)} = \frac{1}{n-2} \mathring{\top} \delta^i \left(\sigma^{q+1} \mathring{\tau}_{ab} \right) \tag{76}$$

on Σ_2

- spacetime with isotropic singularity $\left(\check{M}, \check{g}_{ab}\right)$,

$$\mathring{K}_{ab}^{(i+2)} = \frac{1}{n-2} \mathring{\top} \delta^i \left(\delta \mathring{\chi}_{ab} \right). \tag{77}$$

on Σ_1'



Matching conditions

$$\overline{\mathbf{g}}\big|_{\Sigma_2} = \overline{\mathbf{g}}'\big|_{\Sigma'1}, \quad \mathring{K}^{(j)}_{\mathbf{g}}\big|_{\Sigma_2} = \mathring{K}^{(j)}_{\mathbf{g'}}\big|_{\Sigma'_1}, \quad j = 2, ..., n(?) \qquad (78)$$

Matching of conformal fundamental forms – matching stress-energy tensor densities.

Thank you for your attention!